① $\left(\dfrac{27x^{-12}y^{\frac{3}{2}}}{64x^{18}y^{6}}\right)^{-\frac{2}{3}}$

$= \left(\dfrac{27 \; x^{-12-18} \; y^{\frac{3}{2}-6}}{64}\right)^{-\frac{2}{3}}$

$= \left(\dfrac{27 \; x^{-30} \; y^{-\frac{9}{2}}}{64}\right)^{-\frac{2}{3}}$

$= \left(\dfrac{27}{64x^{30}y^{\frac{9}{2}}}\right)^{-\frac{2}{3}}$

$= \left(\dfrac{64x^{30}y^{\frac{9}{2}}}{27}\right)^{\frac{2}{3}}$

$= \dfrac{64^{\frac{2}{3}} \; x^{20} \; y^{3}}{27^{\frac{2}{3}}}$

$= \dfrac{16 \; x^{20} \; y^{3}}{9}$ //

---

② $\dfrac{\sqrt{25} - \sqrt{-49}}{\sqrt{16} + \sqrt{-81}}$

$= \dfrac{5 - 7i}{4 + 9i}$

$= \dfrac{5 - 7i}{4 + 9i} \cdot \dfrac{4 - 9i}{4 - 9i}$

28
45
-73

$= \dfrac{20 - 73i + 63i^2}{16 - 81i^2}$

$= \dfrac{-43 - 73i}{97}$

$= -\dfrac{43}{97} - \dfrac{73}{97}i$ //

---

③ (i) $b^2 - 4ac = (-5)^2 - 4(3)(8)$

$= 25 - 96$

$= -71$

Two Complex Solutions. //

(ii) $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-(-5) \pm \sqrt{-71}}{2(3)}$

$= \dfrac{5 \pm \sqrt{71}\,i}{6}$

$= \dfrac{5}{6} \pm \dfrac{\sqrt{71}}{6}i$ //

---

④ $\sqrt{5x+6} + \sqrt{3x+4} = 2$

$\sqrt{5x+6} = 2 - \sqrt{3x+4}$

$\left(\sqrt{5x+6}\right)^2 = \left(2 - \sqrt{3x+4}\right)^2$

$5x + 6 = 4 - 4\sqrt{3x+4} + 3x + 4$

$2x - 2 = -4\sqrt{3x+4}$

$(2x-2)^2 = \left(-4\sqrt{3x+4}\right)^2$

$4x^2 - 8x + 4 = 16(3x+4)$

$4x^2 - 8x + 4 = 48x + 64$

$4x^2 - 56x - 60 = 0$

$x^2 - 14x - 15 = 0$

$(x-15)(x+1) = 0$

$x - 15 = 0 \qquad x + 1 = 0$

$x \cancel{=} 15 \qquad x = -1$

$\{-1\}$ //

(5)

$$\frac{5-\sqrt{3x}}{\sqrt{2x}+\sqrt{3y}}$$

$$= \frac{5-\sqrt{3x}}{\sqrt{2x}+\sqrt{3y}} \cdot \frac{\sqrt{2x}-\sqrt{3y}}{\sqrt{2x}-\sqrt{3y}}$$

$$= \frac{5\sqrt{2x} - 5\sqrt{3y} - x\sqrt{6} + 3\sqrt{xy}}{2x-3y} \;//$$

---

(6) $x^{\frac{2}{3}} - 9x^{\frac{1}{3}} + 20 = 0$

let $u = x^{\frac{1}{3}} = \sqrt[3]{x}$

$u^2 - 9u + 20 = 0$

$(u-4)(u-5) = 0$

$u - 4 = 0 \qquad u - 5 = 0$

$\sqrt[3]{x} - 4 = 0 \qquad \sqrt[3]{x} - 5 = 0$

$\sqrt[3]{x} = 4 \qquad \sqrt[3]{x} = 5$

$(\sqrt[3]{x})^3 = (4)^3 \qquad (\sqrt[3]{x})^3 = (5)^3$

$x = 64 \qquad x = 125$

$\{64, 125\}$ //

---

(7) $f(x) = \sqrt{x-2}$

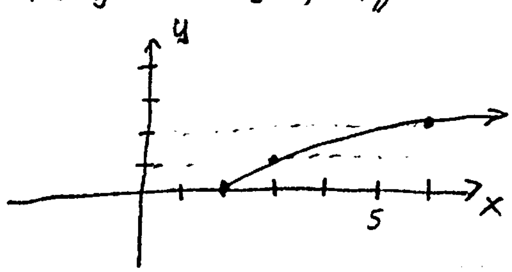
Domain : radicant ≥ 0

$x - 2 \geq 0$

$x \geq 2$

$[2, \infty)$ //

Range : $[0, \infty)$ //



---

(8) $4x^2 + 1 = -6x$

$4x^2 + 6x + 1 = 0$

$x^2 + \frac{3}{2}x + \frac{1}{4} = 0$

$x^2 + \frac{3}{2}x = -\frac{1}{4}$

$x^2 + \frac{3}{2}x + \frac{9}{16} = -\frac{1}{4} + \frac{9}{16}$

$\left(x + \frac{3}{4}\right)^2 = \frac{5}{16}$

$\sqrt{\left(x+\frac{3}{4}\right)^2} = \pm\sqrt{\frac{5}{16}}$

$x + \frac{3}{4} = \pm\frac{\sqrt{5}}{4}$

$x = -\frac{3}{4} \pm \frac{\sqrt{5}}{4}$ //

---

(9) $5\sqrt[4]{32a^9} + 3a\sqrt[4]{162a^5} - 2a^2\sqrt[4]{512a}$

16 2   81 2   256 2

$= 5 \cdot 2a^2\sqrt[4]{2a} + 3a \cdot 3a\sqrt[4]{2a} - 2a^2 \cdot 4\sqrt[4]{2a}$

$= 10a^2\sqrt[4]{2a} + 9a^2\sqrt[4]{2a} - 8a^2\sqrt[4]{2a}$

$= 11a^2\sqrt[4]{2a}$ //

---

(10) $\sqrt{4a+1} - a = -1$

$\sqrt{4a+1} = a - 1$

$(\sqrt{4a+1})^2 = (a-1)^2$

$4a + 1 = a^2 - 2a + 1$

$0 = a^2 - 6a$

$0 = a(a-6)$

$\cancel{a = 0} \qquad a - 6 = 0$

$a = 6$

$\{6\}$ //

(11) $(y^2+1)^2 - 8(y^2+1) - 9 = 0$

Let $u = y^2+1$

$u^2 - 8u - 9 = 0$

$(u-9)(u+1) = 0$

$u - 9 = 0 \qquad u+1 = 0$

$(y^2+1) - 9 = 0 \qquad (y^2+1)+1 = 0$

$y^2 - 8 = 0 \qquad y^2 + 2 = 0$

$y^2 = 8 \qquad y^2 = -2$

$y = \pm\sqrt{8} \qquad y = \pm\sqrt{-2}$

$y = \pm 2\sqrt{2} \qquad y = \pm\sqrt{2}\, i$

$\{\pm\sqrt{2}\, i, \pm 2\sqrt{2}\}$ //

---

(12) $f(x) = 2x^2 + 12x + 17$

vertex : $x = -\frac{b}{2a} = -\frac{12}{2(2)} = -3$

$y = 2(-3)^2 + 12(-3) + 17 = -1$

(i) $f(x) = 2(x+3)^2 - 1$

(ii) vertex : $(-3, -1)$

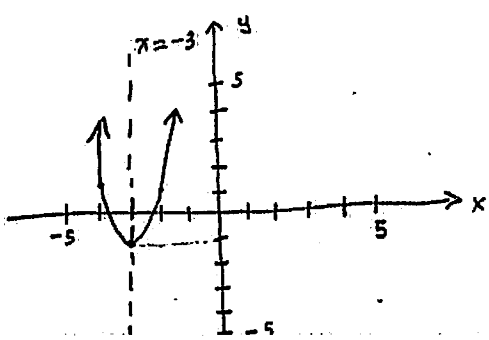
(iii) axis of symmetry : $x = -3$

(iv) min function value $= -1$

(v) domain : $(-\infty, \infty)$

range : $[-1, \infty)$

(vi)



(13) (i) $t = -\frac{b}{2a}$

$= -\frac{155}{2(-16)}$

$= 4.84375$

(ii) $s(4.84375)$

$= -16(4.84375)^2 + 155(4.84375) + 8$

$= 383.390625$

(iii) $0 = -16t^2 + 155t + 8$

$t = \frac{-155 \pm \sqrt{(155)^2 - 4(-16)(8)}}{2(-16)}$

$= \frac{-155 \pm \sqrt{24537}}{-32}$

≈ 9.7 sec.