Math 131 Test 3 Name: 118

Show all necessary steps clearly, neatly, systematically to receive full credit.

1. Perform indicated operation.

2 a. $(5x^2y^2+6x+9y)+(-x-y+x^2y^2)$

$= 5x^2y^2+6x+9y-x-y+x^2y^2$

$= 6x^2y^2+5x+8y$ //

2 b. $(3m^3+7m^2-20)-(-8m+8m^3-14)$

$= 3m^3+7m^2-20+8m-8m^3+14$

$= -5m^3+7m^2+8m-6$ //

2 c. $12x^2(-10x^6+3x^2-9)$

$= -120x^8+36x^4-108x^2$ //

2 d. $(x-1)(4x-7)$

$= 4x^2-7x-4x+7$

$= 4x^2-11x+7$ //

2 e. $(5x-2)^2$

$= 25x^2-20x+4$ //

2 f. $(ab-0.6)(0.8ab+0.4)$

$= 0.8a^2b^2+0.4ab-0.48ab-0.24$

$= 0.8a^2b^2-0.08ab-0.24$ //

2 g. $(9x-7y)(9x+7y)$

$= 81x^2-49y^2$ //

2 h. $(6x-1)(x^2-4x+1)$

$= 6x^3-24x^2+6x-x^2+4x-1$

$= 6x^3-25x^2+10x-1$ //

2 i. $\dfrac{8x^2+24x-13}{4x}$

$= \dfrac{8x^2}{4x} + \dfrac{24x}{4x} - \dfrac{13}{4x}$

$= 2x + 6 - \dfrac{13}{4x}$ //

2 j. Use synthetic division $(6x^2+37x-18)\div(x-7)$

$$\begin{array}{c|cccc} 7 & 6 & 37 & -18 \\ & \downarrow & 42 & 553 \\ \hline & 6 & 79 & 535 \end{array}$$

$6x + 79 + \dfrac{535}{x-7}$ //

4 k. $[y-(x-2)][y+(x-2)]$

$= y^2 - (x-2)^2$

$= y^2 - (x^2 - 4x + 4)$

$= y^2 - x^2 + 4x - 4$ //

4 l. $(x^4+4x-7-3x^2)\div(x^2-4)$

$$\begin{array}{r} x^2 + 1 \\ x^2 - 4 \overline{)\, x^4 + 0x^3 - 3x^2 + 4x - 7} \\ -(\ominus x^4 \quad \oplus 4x^2) \\ \hline x^2 + 4x - 7 \\ -(\ominus x^2 \quad \oplus 4) \\ \hline 4x - 3 \end{array}$$

$x^2 + 1 + \dfrac{4x-3}{x^2-4}$ //

2. Factor the polynomial completely. If the polynomial cannot be factored, say it is prime.

2 a. $xy+11x-2y-22$

$= x(y+11) - 2(y+11)$

$= (y+11)(x-2)$ //

2 b. x^4-1

$= (x^2)^2 - (1)^2$

$= (x^2-1)(x^2+1)$

$= (x-1)(x+1)(x^2+1)$ //

2 c. $x^2-11x+30$

$= (x-5)(x-6)$ //

2 d. $9x^2+13x-10$

$= (9x-5)(x+2)$ //

1 e. $25x^2+49$

$=$ prime //

1 f. $15(x+11)-y(x+11)$

$= (x+11)(15-y)$ //

2 g. $3x^2y - 9xy - 120y$

$= 3y\ (x^2 - 3x - 40)$

$= 3y\ (x-8)(x+5)$ //

2 h. $28x^3 - 132x^2 + 80x$

$= 4x\ (7x^2 - 33x + 20)$

$= 4x\ (7x-5)(x-4)$ //

2 i. $20x^2 + 27x + 9$

$= (4x+3)(5x+3)$ //

2 j. $x^2 - 5x - 24$

$= (x-8)(x+3)$ //

2 k. $25x^3 - 81x$

$= x\ (25x^2 - 81)$

$= x\ (5x-9)(5x+9)$

2 l. $3t^5 - 6t^4 - 45t^3$

$= 3t^3\ (t^2 - 2t - 15)$

$= 3t^3\ (t-5)(t+3)$ //

2 m. $12x^2 + 15x - 20x - 25$

$= 3x\ (4x+5) - 5\ (4x+5)$

$= (4x+5)\ (3x-5)$ //

2 n. $x^2 + 2xy - 15y^2$

$= (x+5y)\ (x-3y)$ //

2 o. $81x^2 + 90xy + 25y^2$

$(9x)^2 \qquad (5y)^2$

$= (9x+5y)^2$ //

2 p. $125x^3 + y^3$

$= (5x)^3 + (y)^3$

$= (5x+y)\ (25x^2 - 5xy + y^2)$

4 q. $4x^4y - 64y^5$

$= 4y(x^4 - 16y^4)$

$= 4y(x^2 - 4y^2)(x^2 + 4y^2)$

$= 4y(x - 2y)(x + 2y)(x^2 + 4y^2)$ //

4 r. $x^3 - 4x^2 - 9x + 36$

$= x^2(x-4) - 9(x-4)$

$= (x-4)(x^2 - 9)$

$= (x-4)(x-3)(x+3)$ //

4 s. $18x^3y + 57x^2y^2 + 30xy^3$

$= 3xy(6x^2 + 19xy + 10y^2)$

$= 3xy(3x + 2y)(2x + 5y)$ //

3. Solve:

4 a. $9x^2 = 16x$

$9x^2 - 16x = 0$

$x(9x - 16) = 0$

$x = 0$ | $9x - 16 = 0$, $x = \frac{16}{9}$

$\{0, \frac{16}{9}\}$ //

4 b. $9x^2 = 16$

$9x^2 - 16 = 0$

$(3x - 4)(3x + 4) = 0$

$3x - 4 = 0$, $x = \frac{4}{3}$ | $3x + 4 = 0$, $x = -\frac{4}{3}$

$\{-\frac{4}{3}, \frac{4}{3}\}$ //

4 c. $x(3x + 8) = -5$

$3x^2 + 8x + 5 = 0$

$(3x + 5)(x + 1) = 0$

$3x + 5 = 0$, $x = -\frac{5}{3}$ | $x + 1 = 0$, $x = -1$

$\{-\frac{5}{3}, -1\}$ //

4 d. $(x+8)(x-3) = -30$

$x^2 + 5x - 24 = -30$

$x^2 + 5x + 6 = 0$

$(x + 3)(x + 2) = 0$

$x + 3 = 0$, $x = -3$ | $x + 2 = 0$, $x = -2$

$\{-3, -2\}$ //

8 e. $(y+1)^2 - 8(y+1) - 9 = 0$

let $u = y + 1$

$u^2 - 8u - 9 = 0$

$(u - 9)(u + 1) = 0$

$u - 9 = 0$, $y + 1 - 9 = 0$, $y - 8 = 0$, $y = 8$ | $u + 1 = 0$, $y + 1 + 1 = 0$, $y + 2 = 0$, $y = -2$

$\{-2, 8\}$ //

3 4. Simplify: $\dfrac{xy+4y-7x-28}{x^2+11x+28}$.

$= \dfrac{(x+4)(y-7)}{(x+7)(x+4)}$

$= \dfrac{y-7}{x+7}$ //

side

$xy+4y-7x-28$

$y(x+4)-7(x+4)$

$(x+4)(y-7)$

5 7. Divide: $\dfrac{x^2-25}{2x-2} \div \dfrac{x^2+10x+25}{x^2+4x-5}$.

$= \dfrac{x^2-25}{2x-2} \cdot \dfrac{x^2+4x-5}{x^2+10x+25}$

$= \dfrac{(x-5)(x+5)}{2(x-1)} \cdot \dfrac{(x+5)(x-1)}{(x+5)(x+5)}$

$= \dfrac{x-5}{2}$ //

5 5. Multiply: $\dfrac{25-y^2}{y^2-2y-35} \cdot \dfrac{y^2-8y-20}{y^2-3y-10}$.

$= \dfrac{(5-y)(5+y)}{(y-7)(y+5)} \cdot \dfrac{(y-10)(y+2)}{(y-5)(y+2)}$

$= -\dfrac{y-10}{y-7}$ //

8 8. Simplify: $\dfrac{\dfrac{2x}{x+1} - \dfrac{x^2-3}{x^2+3x+2}}{4+\dfrac{4}{x+2}}$.

$= \dfrac{\dfrac{2x}{x+1} - \dfrac{x^2-3}{(x+2)(x+1)}}{4+\dfrac{4}{x+2}} \cdot \dfrac{(x+2)(x+1)}{(x+2)(x+1)}$

$= \dfrac{2x \cdot (x+2) - (x^2-3)}{4(x+2)(x+1) + 4(x+1)}$

$= \dfrac{2x^2+4x-x^2+3}{4(x+1)[(x+2)+1]}$

$= \dfrac{x^2+4x+3}{4(x+1)(x+3)}$

$= \dfrac{(x+3)(x+1)}{4(x+1)(x+3)}$

$= \dfrac{1}{4}$ //

5 6. Perform indicated operation: $\dfrac{2x}{x^2-16} - \dfrac{1}{4-x}$.

$= \dfrac{2x}{(x-4)(x+4)} - \dfrac{1}{-(x-4)}$

$= \dfrac{2x}{(x-4)(x+4)} + \dfrac{1}{x-4}$

$= \dfrac{2x+1 \cdot (x+4)}{(x-4)(x+4)}$

$= \dfrac{2x+x+4}{(x-4)(x+4)}$

$= \dfrac{3x+4}{(x-4)(x+4)}$ //

5 9. Consider the polynomial function $f(x)=2x^2-5x+4$, **(i)** find $f(-3)$, **(ii)** find $f(x+h)$ and simplify your final answer.

i) $f(-3) = 2(-3)^2 - 5(-3) + 4$

$= 18 + 15 + 4$

$= 37$ //

ii) $f(x+h) = 2(x+h)^2 - 5(x+h) + 4$

$= 2(x^2+2xh+h^2) - 5x - 5h + 4$

$= 2x^2 + 4xh + 2h^2 - 5x - 5h + 4$ //

5 10. Let $f(x)=x^4-65x^2+55$. Use the Factor Theorem to determine whether $x-8$ is a factor of the function.

8	1	0	−65	0	55
	↓	8	64	−8	−64
	1	8	−1	−8	−9

Since remainder is not 0, by the factor theorem $x-8$ is not a factor of the function.

8 11. The base of a triangle is 4 m shorter than its height. What are the height and base of the triangle if its area is 48 square meters?

① base $= x-4$

height $= x$

② $48 = \frac{1}{2}(x-4)\cdot x$

$48 = \frac{1}{2}x(x-4)$

$48 = \frac{1}{2}x^2 - 2x$

$0 = \frac{1}{2}x^2 - 2x - 48$

$0 = x^2 - 4x - 96$

$0 = (x-12)(x+8)$

$x-12=0$ | $x+8=0$

$x=12$ | $x=-8$

③ base $= 8$ m

height $= 12$ m

18