Math 7A Test 3 Name: 110

Show all necessary steps clearly, neatly, systematically to receive full credit.

1. (5 pts) Write $\ln\left[\frac{5x^2\sqrt[3]{1-x}}{4(x+1)^2}\right]^2$ as a sum and difference of logarithms. Express all powers as factors.

$= 2\left[\ln 5 + 2\ln x + \frac{1}{3}\ln(1-x) - \ln 4 - 2\ln(x+1)\right]$

$= 2\ln 5 + 4\ln x + \frac{2}{3}\ln(1-x) - 2\ln 4 - 4\ln(x+1)$ //

2. (5 pts) Write $\frac{1}{3}\log(x-2) - \frac{2}{3}\log(x+4) + \frac{1}{3}\log(x+1) - \frac{2}{3}\log\left(\frac{4}{x}\right)$ as a single logarithm.

$= \frac{1}{3}\left[\log(x-2) + \log(x+1) - 2\log(x+4) - 2\log\left(\frac{4}{x}\right)\right]$

$= \log\left[\frac{(x-2)(x+1)}{(x+4)^2\left(\frac{4}{x}\right)^2}\right]^{\frac{1}{3}}$ //

3. (5 pts) Solve: $-1 + 9\log_4(7x+8) = 17$.

$9\log_4(7x+8) = 18$

$\log_4(7x+8) = 2$

$7x+8 = 4^2$

$7x+8 = 16$

$7x = 8$

$x = \frac{8}{7}$

$\left\{\frac{8}{7}\right\}$ //

4. (5 pts) Solve: $9^{2x} \cdot 27^{x^2} = 3^{-1}$.

$(3^2)^{2x} \cdot (3^3)^{x^2} = 3^{-1}$

$3^{4x+3x^2} = 3^{-1}$

$3x^2 + 4x = -1$

$3x^2 + 4x + 1 = 0$

$(3x+1)(x+1) = 0$

$3x+1 = 0$ | $x+1 = 0$

$x = -\frac{1}{3}$ | $x = -1$

$\left\{-1, -\frac{1}{3}\right\}$ //

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5. (5 pts) Solve: $4^{3x+3} \cdot 64^{-x} = 1$.

$4^{3x+3} \cdot 4^{-3x} = 4^{0}$

$4^{3x+3-3x} = 4^{0}$

$4^{3} = 4^{0}$

ϕ //

6. (5 pts) Let $f(x) = \left(\frac{1}{16}\right)^{x}$ and $g(x) = \left(\frac{1}{4}\right)^{x-2}$.

Solve $\frac{f(x)}{g(x)} = 32$.

$\frac{\left(\frac{1}{16}\right)^{x}}{\left(\frac{1}{4}\right)^{x-2}} = 32$

$\frac{2^{-4x}}{2^{-2(x-2)}} = 2^{5}$

$2^{-4x+2(x-2)} = 2^{5}$

$-4x + 2x - 4 = 5$

$-2x = 9$

$x = -\frac{9}{2}$

$\left\{-\frac{9}{2}\right\}$ //

7. (5 pts) Let $f(x) = a^{x}$, show that $f(A-B) = \frac{f(A)}{f(B)}$.

L.H.S $= f(A-B)$

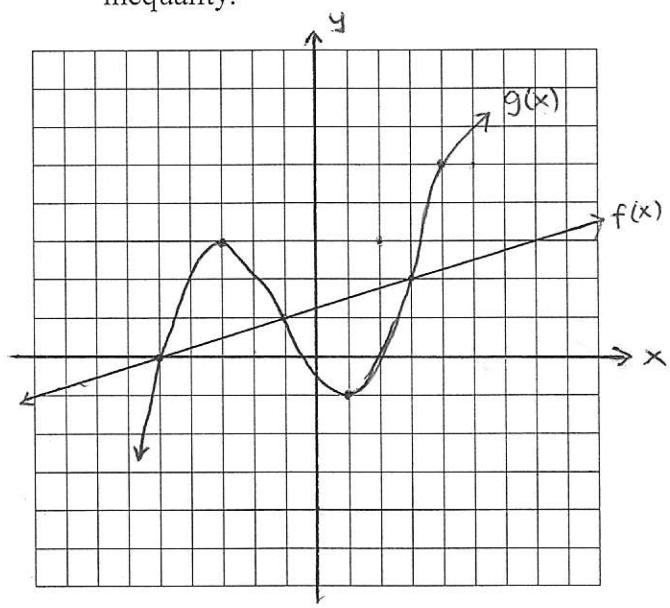
$= a^{A-B}$

$= \frac{a^{A}}{a^{B}}$

$= \frac{f(A)}{f(B)}$

$=$ R.H.S. //

8. (5 pts) Consider the graph and solve the following inequality.



a. $f(x) > 0$. $(-5, \infty)$

b. $g(x) < 0$. $(-\infty, -5) \cup (-\frac{1}{2}, 2)$

c. $g(x) \geq f(x)$. $[-5, -1] \cup [3, \infty)$

d. $f(x) < g(x)$. $(-5, -1) \cup (3, \infty)$

e. $f(x) \geq g(x)$. $(-\infty, -5] \cup [-1, 3]$

9. (7 pts) Let $f(x) = 6x^2 - 2x$ and $g(x) = 6 + 3x$. Solve $f(x) < g(x)$.

$f(x) < g(x)$

$6x^2 - 2x < 6 + 3x$

$6x^2 - 5x - 6 < 0$

$(3x+2)(2x-3) < 0$

zeros

$3x + 2 = 0$ | $2x - 3 = 0$

$x = -\frac{2}{3}$ | $x = \frac{3}{2}$

I II III

pick $\boxed{-1}$ pick $\boxed{0}$ pick $\boxed{2}$

$+ \quad -\frac{2}{3} \quad - \quad \frac{3}{2} \quad +$

$\left(-\frac{2}{3}, \frac{3}{2}\right)$ //

10. (7 pts) Find the inverse function of given one-to-one function: $f(x) = \dfrac{3x+4}{2x-3}$.

$y = \dfrac{3x+4}{2x-3}$

$x = \dfrac{3y+4}{2y-3}$

$x(2y-3) = 3y+4$

$2xy - 3x = 3y + 4$

$2xy - 3y = 3x + 4$

$y(2x-3) = 3x+4$

$y = \dfrac{3x+4}{2x-3}$

$f^{-1}(x) = \dfrac{3x+4}{2x-3}$ //

11. (7 pts) Find a quadratic function $f(x) = ax^2 + bx + c$ whose vertex is (- 1 , - 9) and x-intercept (- 4 , 0).

$f(x) = a(x-h)^2 + k$

$f(x) = a(x+1)^2 - 9$

$\langle f(-4) = 0 \rangle$

$0 = a(-4+1)^2 - 9$

$9 = a(-3)^2$

$9 = 9a$

$1 = a$

$f(x) = 1(x+1)^2 - 9$

$f(x) = x^2 + 2x + 1 - 9$

$= x^2 + 2x - 8$ //

$\boxed{21}$

12. (7 pts) Let $f(x) = 4x^2 - 8x + 3$.

a. Find vertex.

$x = -\frac{b}{2a} = -\frac{-8}{2(4)} = 1$

$y = f(1) = 4(1)^2 - 8(1) + 3 = -1$

$(1, -1)$

b. Find axis of symmetry.

$x = 1$

c. Write in $f(x) = a(x-h)^2 + k$ form.

$f(x) = 4(x-1)^2 - 1$

d. Find maximum or minimum function value.

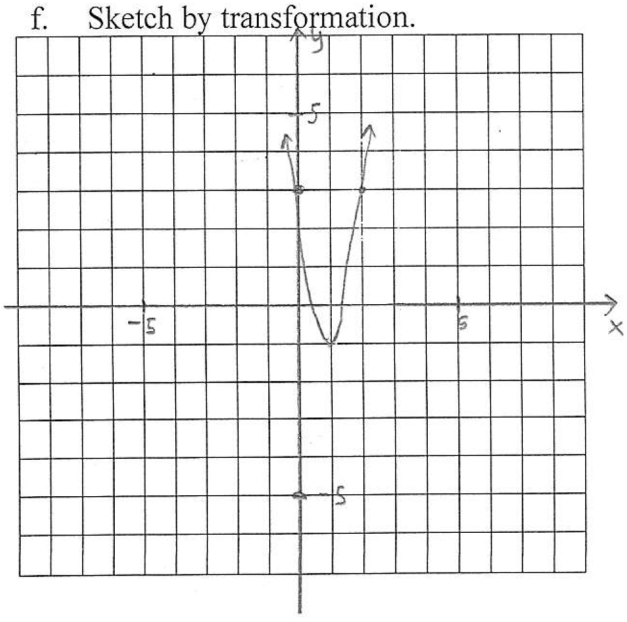
minimum function value $= -1$

e. Find increasing and decreasing interval.

Increasing: $(1, \infty)$

Decreasing: $(-\infty, 1)$

f. Sketch by transformation.



13. (7 pts) A ball is thrown vertically upward with an initial velocity of 96 feet per second. The distance s (in feet) of the ball from the ground after t seconds is $s(t) = 96t - 16t^2$.

a. When will the ball reach the maximum height?

$t = -\frac{b}{2a}$

$= -\frac{96}{2(-16)}$

$= 3$

After 3 seconds.

b. What is its maximum height?

$s(3) = 96(3) - 16(3)^2$

$= 288 - 144$

$= 144$

maximum height is 144 ft.

c. When will it strike the ground?

$\langle s(t) = 0, t = ? \rangle$

$0 = 96t - 16t^2$

$0 = 16t(6 - t)$

$16t = 0 \quad | \quad 6 - t = 0$

$t = 0 \quad | \quad 6 = t$

After 6 seconds.

d. For what time is the ball more than 128 feet above the ground?

$\langle s(t) > 128, t = ? \rangle$

$96t - 16t^2 > 128$

$t^2 - 6t < -8$

$t^2 - 6t + 8 < 0$

$(t-4)(t-2) < 0$

zero

$t - 4 = 0 \quad | \quad t - 2 = 0$

$t = 4 \quad | \quad t = 2$

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$t: (2, 4)$

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$(-\infty,\infty)$ $[2,\infty)$ $(-\infty,5)\cup(5,\infty)$ $(-1,\infty)$

14. Let $f(x)=x^2+4$, $g(x)=\sqrt{x-2}$, $h(x)=\dfrac{x+4}{x-5}$, $k(x)=3\log_3(x+1)$.

a. Find domain of $.(h\circ f)(x)$.

$$(h\circ f)(x) = h(f(x)) = h(x^2+4) = \frac{x^2+4+4}{(x^2+4)-5} = \frac{x^2+8}{x^2-1} = \frac{x^2+8}{(x-1)(x+1)}$$

D of f : $(-\infty,\infty)$

D of $\dfrac{x^2+8}{(x-1)(x+1)}$: $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$

D of $(h\circ f)(x)$: $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$ //

b. Find domain of $.(g\circ f)(x)$.

$$(g\circ f)(x) = g(f(x)) = g(x^2+4) = \sqrt{(x^2+4)-2} = \sqrt{x^2+2}$$

D of f : $(-\infty,\infty)$

D of $\sqrt{x^2+2}$: $(-\infty,\infty)$

D of $(g\circ f)(x)$: $(-\infty,\infty)$ //

c. Find domain of $.(h\circ g)(x)$.

$$(h\circ g)(x) = h(g(x)) = h(\sqrt{x-2}) = \frac{\sqrt{x-2}+4}{\sqrt{x-2}-5}$$

D of g : $[2,\infty)$

D of $\dfrac{\sqrt{x-2}+4}{\sqrt{x-2}-5}$:

$\sqrt{x-2}-5 \neq 0$ | $x-2 \geq 0$

$\sqrt{x-2} = 5$ | $x \geq 2$

$x-2 = 25$

$x = 27$

$[2,27)\cup(27,\infty)$

D of $(h\circ g)(x)$: $[2,27)\cup(27,\infty)$ //

d. Find domain of $.(g\circ h)(x)$.

$$(g\circ h)(x) = g(h(x)) = g\left(\frac{x+4}{x-5}\right) = \sqrt{\frac{x+4}{x-5}-2}$$

D of h : $(-\infty,5)\cup(5,\infty)$

D of $\sqrt{\frac{x+4}{x-5}-2}$:

$\dfrac{x+4}{x-5}-2 \geq 0$

$\dfrac{x+4-2x+10}{x-5} \geq 0$

$\dfrac{-x+14}{x-5} \geq 0$

$-$ 5 $+$ 14 $-$

$(5,14]$

D of $(g\circ h)(x)$: $(5,14]$ //

e. Find domain of $.(k\circ h)(x)$.

$$(k\circ h)(x) = k(h(x)) = k\left(\frac{x+4}{x-5}\right) = 3\log_3\left(\frac{x+4}{x-5}+1\right)$$

D of h : $(-\infty,5)\cup(5,\infty)$

D of $3\log_3\left(\frac{x+4}{x-5}+1\right)$:

$\dfrac{x+4}{x-5}+1 > 0$

$\dfrac{x+4+x-5}{x-5} > 0$

$\dfrac{2x-1}{x-5} > 0$

$+$ $\frac{1}{2}$ $-$ 5 $+$

$(-\infty,\frac{1}{2})\cup(5,\infty)$

D of $(k\circ h)(x)$: $(-\infty,\frac{1}{2})\cup(5,\infty)$ //

f. Find domain of $.(h\circ g\circ f)(1)$.

$$(h\circ g\circ f)(1) = h(g(f(1))) = h(g(5)) = h(\sqrt{3}) = \frac{\sqrt{3}+4}{\sqrt{3}-5}$$

side

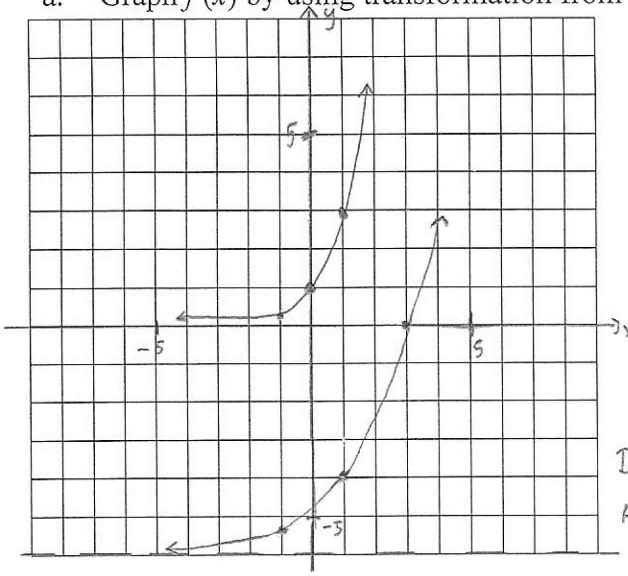
$f(1) = 1^2+4 = 5$

$g(5) = \sqrt{5-2} = \sqrt{3}$

$h(\sqrt{3}) = \dfrac{\sqrt{3}+4}{\sqrt{3}-5}$

15. Let $f(x) = 2 \cdot 3^{\frac{1}{2}(x-1)} - 6$.

a. Graph $f(x)$ by using transformation from $f(x) = 3^x$. State the domain and range.



① $y = 3^x$

② $y = 3^{\frac{1}{2}x}$

④ $y = 2 \cdot 3^{\frac{1}{2}x}$

⑤ $y = 2 \cdot 3^{\frac{1}{2}(x-1)}$

⑥ $y = 2 \cdot 3^{\frac{1}{2}(x-1)} - 6$

D : $(-\infty, \infty)$

R : $(-6, \infty)$

Solving method

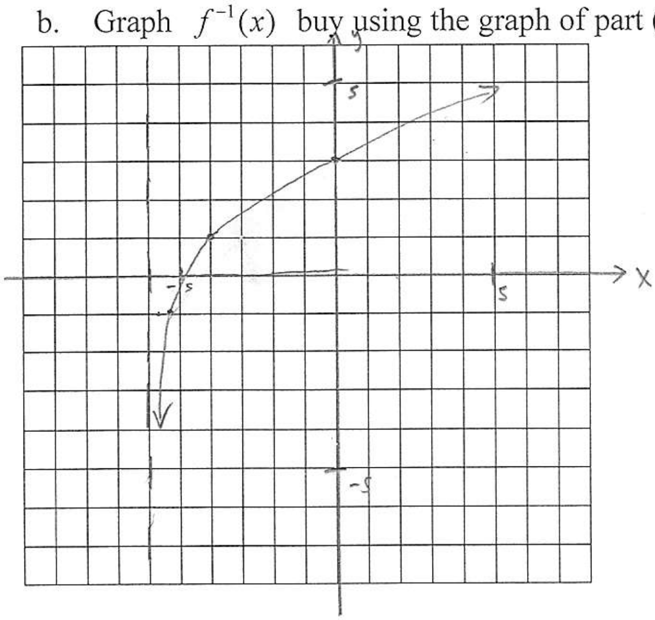
horizontal stretching and shifting

$\frac{1}{2}(x-1) = 1$
$x - 1 = 2$
$x = 3$

$\frac{1}{2}(x-1) = 0$
$x - 1 = 0$
$x = 1$

$\frac{1}{2}(x-1) = -1$
$x - 1 = -2$
$x = -1$

b. Graph $f^{-1}(x)$ buy using the graph of part (a). State the domain and range.



D : $(-6, \infty)$

R : $(-\infty, \infty)$