Math 7A Test 4 Name:____________________

Show all necessary steps clearly, neatly, and systematically to receive full credit.

1. (3 pts) Convert $20^\circ 08' 12''$ to a decimal in degree. Round to two decimal places.

$$= 20^\circ + 8' \cdot \frac{1^\circ}{60'} + 12'' \cdot \frac{1'}{60''} \cdot \frac{1^\circ}{60'}$$

$$\approx 20.14$$

2. (3 pts) Convert 20.812° to $D^\circ M' S''$ form. Round to the nearest second.

$$0.812^\circ \cdot \frac{60'}{1^\circ} = 48.72'$$

$$0.72' \cdot \frac{60''}{1'} = 43.2''$$

$$20^\circ\ 48'\ 43''$$

3. (3 pts) Convert 215° to radians.

$$215^\circ \cdot \frac{\pi}{180}$$

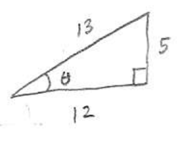
$$= \frac{43\pi}{36} \text{ radians.}$$

4. (3 pts) Convert $-\frac{11\pi}{6}$ to degrees.

$$-\frac{11\cancel{\pi}}{\cancel{6}_{1}} \cdot \frac{\cancel{180}^{30}}{\cancel{\pi}}$$

$$= -330^\circ$$

5. (5 pts) Find the remaining five trigonometric functions of the acute angle θ : $\csc\theta = \frac{13}{5}$.



$$\csc\theta = \frac{13}{5}$$

$$a^2 + b^2 = c^2$$

$$5^2 + b^2 = 13^2$$

$$25 + b^2 = 169$$

$$b^2 = 144$$

$$b = 12$$

$$\sin\theta = \frac{5}{13}$$

$$\cos\theta = \frac{12}{13}$$

$$\tan\theta = \frac{5}{12}$$

$$\sec\theta = \frac{13}{12}$$

$$\cot\theta = \frac{12}{5}$$

6. (3 pts each) Use the identities and theorem to find exact value.

a. $\tan 35^\circ \sec 55^\circ \cos 35^\circ$

$= \tan 35^\circ \cdot \csc(90^\circ - 55^\circ) \cdot \cos 35^\circ$

$= \tan 35^\circ \cdot \csc 35^\circ \cdot \cos 35^\circ$

$= \dfrac{\sin 35^\circ}{\cos 35^\circ} \cdot \dfrac{1}{\sin 35^\circ} \cdot \cos 35^\circ$

$= 1$ //

b. $\cos 35^\circ \sin 55^\circ + \cos 55^\circ \sin 35^\circ$

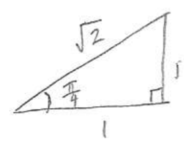
$= \cos 35^\circ \cos(90^\circ - 55^\circ) + \sin(90^\circ - 55^\circ) \sin 35^\circ$

$= \cos 35^\circ \cos 35^\circ + \sin 35^\circ \sin 35^\circ$

$= \cos^2 35^\circ + \sin^2 35^\circ$

$= 1$ //

7. (5 pts) Find the exact value.



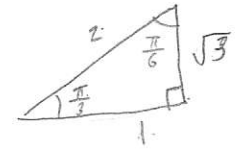
a. $\sin^2\left(\dfrac{\pi}{4}\right) \div \tan\left(\dfrac{\pi}{6}\right) \cdot \sec\left(\dfrac{\pi}{3}\right)$

$= \left(\dfrac{1}{\sqrt{2}}\right)^2 \div \dfrac{1}{\sqrt{3}} \cdot \dfrac{2}{1}$

$= \dfrac{1}{2} \div \dfrac{1}{\sqrt{3}} \cdot \dfrac{2}{1}$

$= \dfrac{1}{2} \cdot \dfrac{\sqrt{3}}{1} \cdot \dfrac{2}{1}$

$= \sqrt{3}$ //



b. $\csc^2\left(\dfrac{\pi}{4}\right) \cos 60^\circ + \tan^\circ\left(\dfrac{\pi}{3}\right)$

$= \left(\dfrac{\sqrt{2}}{1}\right)^2 \cdot \dfrac{1}{2} + \dfrac{\sqrt{3}}{1}$

$= \dfrac{2}{1} \cdot \dfrac{1}{2} + \sqrt{3}$

$= 1 + \sqrt{3}$ //

8. (5 pts) Solve: $\log_6(x+3) = 1 - \log_6(x+4)$.

$\log_6(x+3) + \log_6(x+4) = 1$

$\log_6(x+3)(x+4) = 1$

$(x+3)(x+4) = 6^1$

$x^2 + 7x + 12 = 6$

$x^2 + 7x + 6 = 0$

$(x+6)(x+1) = 0$

$x + 6 = 0 \quad | \quad x + 1 = 0$

$x \neq -6 \quad | \quad x = -1$

$\{-1\}$ //

9. (5 pts) Solve: $16^x + 4^{x-1} - 3 = 0$.

$$(4^x)^2 + 4^{-1} \cdot 4^x - 3 = 0$$

let $u = 4^x$

$$u^2 + \frac{1}{4}u - 3 = 0$$

$$4u^2 + u - 12 = 0$$

$$u = \frac{-1 \pm \sqrt{1+192}}{8}$$

$$u = \frac{-1 \pm \sqrt{193}}{8}$$

$$u = \frac{-1 - \sqrt{193}}{8}$$

$$4^x = \frac{-1 - \sqrt{193}}{8}$$

$$\phi$$

$$u = \frac{-1 + \sqrt{193}}{8}$$

$$4^x = \frac{-1 + \sqrt{193}}{8}$$

$$\ln 4^x = \ln\left(\frac{-1+\sqrt{193}}{8}\right)$$

$$x = \frac{\ln\left(\frac{-1+\sqrt{193}}{8}\right)}{\ln(4)}$$

$$x \approx 0.344$$

10. (3,3,6 pts) A circle with radius 32 yards.

a. Find the length of the arc of the circle of radius *r* subtended by the central angle $\theta = 225^\circ$. Round to two decimal places.

$r = 32$ yrds.

$$\theta = 225^\circ = 225 \times \frac{\pi}{180} = \frac{5\pi}{4}$$

$$S = r\theta = \overset{8}{\cancel{32}} \cdot \frac{5\pi}{\cancel{4}_1} = 40\pi \approx 125.66$$

b. Find the area of the sector of the circle of radius *r* formed by the central angle $\theta = 225^\circ$. Round to two decimal places.

$r = 32$ yrds

$\theta = \frac{5\pi}{4}$

$$A = \frac{1}{2} r^2 \theta = \frac{1}{2}(32)^2 \cdot \frac{5\pi}{4} = \frac{1}{2} \cdot 1024 \cdot \frac{5\pi}{4}$$

$$= 128 \cdot 5\pi = 640\pi \text{ yrd}^2 \approx 2010.62$$

c. If an object takes 5 minutes to travel the distance in part a, find its linear speed and angular speed.

$S = 40\pi$

$t = 5$

$$V = \frac{S}{t} = \frac{40\pi}{5} = 8\pi \text{ yrd/mins} \approx 25.13$$

$$\omega = \frac{\theta}{t} = \frac{\frac{5\pi}{4}}{5} = \frac{5\pi}{4} \cdot \frac{1}{5} = \frac{\pi}{4} \text{ radians/mins} \approx 0.79$$

11. (8 pts) How many years will it take for an initial investment of \$25000 to grow to \$80000? Assume a rate of interest of 7% compounded quarterly.

$P = 25000$

$A = 80000$

$r = 0.07$

$n = 4$

$t = ?$

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$80000 = 25000\left(1+\frac{0.07}{4}\right)^{4t}$$

$$3.2 = 1.0175^{4t}$$

$$\ln 3.2 = \ln 1.0175^{4t}$$

$$\ln 3.2 = 4t \cdot \ln 1.0175$$

$$\frac{\ln 3.2}{4\ln 1.0175} = t$$

$$16.76 \approx t$$

12. (8 pts) Reacting with water in acidic solution at $35^{\circ}C$, sucrose $(C_{12}H_{22}O_{11})$ decomposes into glucose $C_6H_{12}O_6$ and fructose $(C_6H_{12}O_6)$ according to the law of uninhibited decay. An initial amount of 0.40 M of sucrose decomposes to 0.36 M in 30 minutes. How much sucrose will remain after 2 hours? How long will it take until 0.10 M of sucrose remains?

$A_0 = 0.40$

$A = 0.36$

$t = 30$

$$A = A_0 e^{kt}$$

$$0.36 = 0.40 e^{30k}$$

$$0.9 = e^{30k}$$

$$\ln 0.9 = \ln e^{30k}$$

$$\ln 0.9 = 30k$$

$$\frac{\ln 0.9}{30} = k$$

$t = 2$ hrs $= 120$ min

$$A = 0.40 e^{\frac{\ln 0.9}{30}\cdot 120}$$

$$A = 0.40 e^{4\ln 0.9}$$

$$A \approx 0.26$$

$A = 0.10$, $t = ?$

$$0.10 = 0.4 e^{\frac{\ln 0.9}{30}\cdot t}$$

$$0.25 = e^{\frac{\ln 0.9}{30} t}$$

$$\ln 0.25 = \ln e^{\frac{\ln 0.9}{30} t}$$

$$\ln 0.25 = \frac{\ln 0.9}{30}\cdot t$$

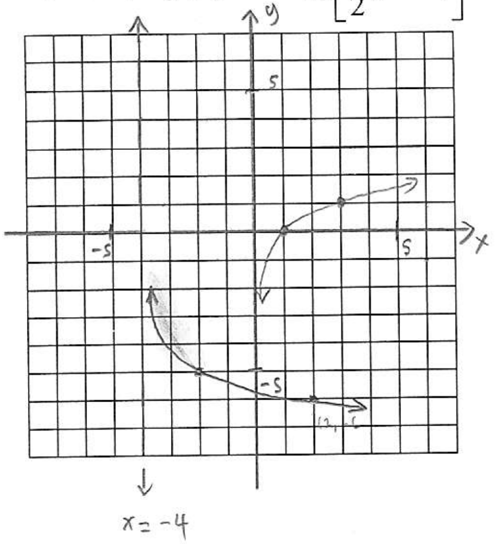
$$\frac{30 \ln 0.25}{\ln 0.9} = t$$

394.7 mins $\approx t$

6.58 hrs

13. (5,3 pts) Graph by transformation using $f(x) = \log_3 x$.

a. Graph $f(x) = -\log_3\left[\frac{1}{2}(x+4)\right] - 5$. Label asymptote.



$y = \log_3 x$

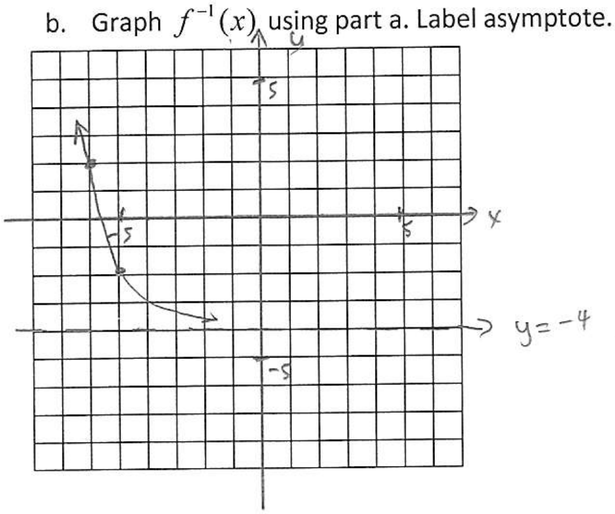
$y = \log_3 \frac{1}{2}x$

$y = -\log_3 \frac{1}{2}x$

$y = -\log_3 \frac{1}{2}(x+4)$

$y = -\log_3 \left[\frac{1}{2}(x+4)\right] - 5$

b. Graph $f^{-1}(x)$ using part a. Label asymptote.



$(-\infty,-1)\cup(-1,\infty)$ $\quad$ $(-1,\infty)$ $\quad$ $[-3,\infty)$

14. (5 pts each) Let $f(x)=\dfrac{x}{x+1}$, $g(x)=\log(x+1)$, $h(x)=\sqrt{x+3}$.

a. Find domain of $(f\circ h)(x)$.

$$(f\circ h)(x) = f(h(x)) = f\left(\sqrt{x+3}\right) = \frac{\sqrt{x+3}}{\sqrt{x+3}+1}$$

D of h : $[-3,\infty)$

D of $\dfrac{\sqrt{x+3}}{\sqrt{x+3}+1}$: $[-3,\infty)$ because $\sqrt{x+3}+1>0$, so only need to consider $\sqrt{x+3}$.

D of $(f\circ h)(x)$:

$[-3,\infty)$ //

b. Find domain of $(g\circ f)(x)$.

$$(g\circ f)(x) = g(f(x)) = g\left(\frac{x}{x+1}\right) = \log\left(\frac{x}{x+1}+1\right)$$

D of f : $(-\infty,-1)\cup(-1,\infty)$

D of $\log\left(\frac{x}{x+1}+1\right)$: $(-\infty,-1)\cup(-\frac{1}{2},\infty)$

$$\frac{x}{x+1}+1>0$$

$$\frac{x+x+1}{x+1}>0$$

$$\frac{2x+1}{x+1}>0$$

✓ ∘ X ∘ ✓

-1 $\quad$ $-\frac{1}{2}$

D of $(g\circ f)(x)$:

$(-\infty,-1)\cup(-\frac{1}{2},\infty)$ //

c. Find domain of $(h\circ f)(x)$.

$$(h\circ f)(x) = h(f(x)) = h\left(\frac{x}{x+1}\right) = \sqrt{\frac{x}{x+1}+3}$$

D of f : $(-\infty,-1)\cup(-1,\infty)$

D of $\sqrt{\frac{x}{x+1}+3}$: $(-\infty,-1]\cup[-\frac{3}{4},\infty)$

$$\frac{x}{x+1}+3\geq 0$$

$$\frac{x+3x+3}{x+1}\geq 0$$

$$\frac{4x+3}{x+1}\geq 0$$

✓ • X • ✓

-1 $\quad$ $-\frac{3}{4}$

D of $(h\circ f)(x)$:

$(-\infty,-1)\cup[-\frac{3}{4},\infty)$ //