95%

Fall 2018 Math 227 Test 4

92/100

Name: Hanlin Meng

Show your work clearly, neatly, systematically and understandably. Round the decimal for probability to 5-decimal place and round the percentage to 3-decimal. Use proper notation. 107 points available.

06, 41, 77

1. (7) A random sample of 160 households receiving the free sample of cereal showed that 45 of them purchased the cereal after. Construct a 93%-CI for the true proportion of households receiving free samples which later purchase the cereal.

1) $\hat{p} = \frac{x}{n} = \frac{45}{160} = 0.28125$

$z = \text{Invnorm}(0.965) = 1.81191$ ✓

2). $E = z_c \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.81191 \cdot \sqrt{\frac{0.28125 \cdot 0.71875}{160}}$

$\hat{q} = 1 - \hat{p} = 1 - 0.28125 = 0.71875$

$= 0.06440$

3). A 93%-CI for p: ✓

$\hat{p} - E < p < \hat{p} + E$

$0.28125 - 0.06440 < p < 0.28125 + 0.06440$

$0.21685 < p < 0.34565$

2. (5) To estimate the president's approval rating, how many people should be sampled so that the margin of error is 1% with 98% confidence level?

$\hat{p}$ is not provided. $\hat{p} = 0.5$

$E = 0.01$. $CL = 98\%$

$z_c = \text{Invnorm}(0.99) = 2.32635$ ✓

$n = \left\lceil \frac{z_c^2 \hat{p}\hat{q}}{E^2} \right\rceil = \left\lceil \frac{2.32635^2 \cdot 0.5 \cdot 0.5}{0.01^2} \right\rceil$

$= \lceil 13529.76081 \rceil = 13530$ ✓

3. (5) Suppose you want to estimate the average number of songs college students store on their portable devices with margin of error to be no more than 5 songs. At 97% confidence level, how many students should you sample if σ = 3.4?

$E = 5$

$CL = 97\%$

$z_c = \text{Invnorm}(0.985) = 2.17009$ ✓

$\sigma = 3.4$

$n = \left\lceil \frac{z_c^2 \cdot \sigma^2}{E^2} \right\rceil = \left\lceil \frac{2.17009^2 \cdot 3.4^2}{5^2} \right\rceil$

$= \lceil 2.17758 \rceil = 3$ ✓

49
43

+17

4. (22:8,7,9) A random sample of 12 obituary notices in the Hawaiian newspapers gave the following information about life span in years of Hawaii residents:

72 68 81 93 56 83 84 97 75 71 86 47

a. Find the sample mean and sample deviation.

$\sum x = 72+68+81+93+56+83+84+97+75+71+86+47 = 913$ ✓

$\bar{x} = \frac{\sum x}{n} = \frac{913}{12} = 76.08333$ ✓

$\sum x^2 = 72^2+68^2+81^2+93^2+56^2+83^2+84^2+97^2+75^2+71^2+86^2+47^2 = 71779$ ✓

$(\sum x)^2 = 833569$

$s^2 = \frac{n\sum x^2 - (\sum x)^2}{n(n-1)} = \frac{12 \cdot 71779 - 833569}{12 \times 11} = 210.44697$ $s = \sqrt{210.44697} = 14.50679$ ✓

b. Assuming that the population life span is normally distributed, construct a 96%-CI for the true standard deviation of the life span of Hawaiian residents?

$-1\frac{1}{2}$

$CL = 96\%$ $df = 11$ Use 95% 1) $\chi^2_R = 21.920$ 2) $\chi^2_L = 3.816$

3) 96%-CI for σ^2: $\frac{11 \cdot 210.44697}{21.920} < \sigma^2 < \frac{11 \cdot 210.44697}{3.816}$

$105.60751 < \sigma^2 < 2314.9667$

96%-CI for σ: $10.27655 < \sigma < 48.1141$

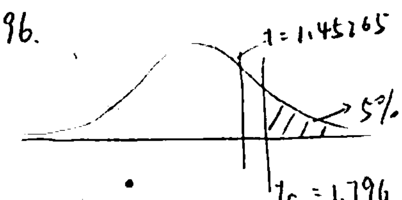
c. Assuming that the population life span is normally-distributed, does this sample indicate that the population mean life span for Hawaii residents is more than 70 years? Test at $\alpha = 0.05$

1) Claim that the population mean life span for Hawii residents is more than 70 years.

2). Hypothesis: $H_0: \mu \le 70$
$H_1: \mu > 70$ (claim. right-tail)

3). Test-stastics: $t = \frac{76.08333 - 70}{\frac{14.50679}{\sqrt{12}}} = 1.45265$

4) Critical Value: $df = 11$ $\alpha = 0.05$ $t_c = 1.796$



5) Informal Conclusion: Fail to reject H_0. ✓

6) At 5%-SL. There's insuffient evidence to support the claim that the population mean lifespan for Hawii residents is more than 70 years

+20.5

5. (9) Let the age of a Quebec resident at the time of the first marriage be normally-distributed. Suppose that a recent study of age at first marriage for a random sample of 46 residents in Quebec gave a variance of 3.4 years. Test the claim that the current variance is less than 4.5 years. Use $\alpha = 0.05$.

$n = 46.\ s^2 = 3.4$

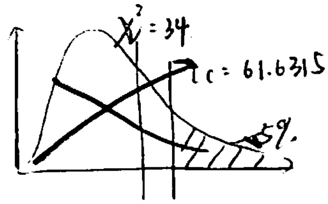
1) Claim that the current variance is less than 45 years.

2) Hypothesis: $H_0: \sigma^2 \geq 4.5$
$H_1: \sigma^2 < 45$ (claim. left-tail)

3) Test-statistic: $\chi_c^2 = \frac{45 \cdot 3.4}{4.5} = 34$

4). Critical Value: $\alpha = 0.05$ df $= 45$

$\chi_c = \cancel{\frac{55.758 + 67.505}{2}} = 61.6315$



5) Informal Conclusion: Fail to reject H_0.

6). At 5%-SI: there's insuffient evidence to support the claim that The current variance is less than 45 years.

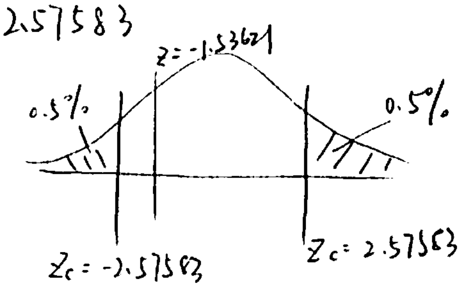
6. (9) "If you could start you career all over again, would you still choose to be a college professor?" About 75% of all US college professors responded "yes" to this question. A random sample of 87 college professors in Colorado showed that 60 claim they would choose college teaching again. Does this indicate that the proportion of professors in Colorado who choose the career again is different from 76%. Use $\alpha = 0.01$ $\hat{p} = \frac{X}{n} = \frac{60}{87} = 0.68966$

1). Claim that the proportion of professors in Colorado who choose the career again is different from 76%,

2) Hypotlesis: $H_0: p = 0.76$
$H_1: p \neq 0.76$ (claim. two-tail)

3) Test-statistic: $z = \frac{0.68966 - 0.76}{\sqrt{\frac{0.76 \cdot 0.24}{87}}} = -1.53621$

4). Critical Value: $z_c = \pm$Invnorm$(0.005) = \pm 2.57583$



5). Informal Conclusion: fail to reject H_0, ✓

6). At 1%-SI. there's insuffient evidence to support the claim that The proportion of professorsin Coloradoo who choose the career again is different from 76%.

+16

7. (21:7,7,7) A mean arsenic level of 8.0 parts per billion (ppb) is considered safe for agricultural use. A well in Texas is used to water cotton crops and is tested on a regular basis for arsenic. A random sample of 30 tests gave a mean of 7.4 ppb arsenic with a standard deviation of 1.7 ppb. Assume that arsenic concentration is normally distributed.

a. Find the 95% confidence interval for the mean arsenic level of that well. CL=95%

1) Best expect for μ: $\bar{X} = 7.4$ $df = 29$. $t_c = 2.045$

2) $E = \frac{t_c \cdot s}{\sqrt{n}} = \frac{2.045 \cdot 1.7}{\sqrt{30}} = 0.63472$.

3) 95%-CI for μ: $\bar{X} - E < \mu < \bar{X} + E$

$7.4 - 0.63472 < \mu < 7.4 + 0.63472$

$6.76528 < \mu < 8.03472$ ✓

b. If the actual standard deviation of the arsenic level of that well is 2.0 ppb. Find the 95% confidence interval for the mean arsenic level of that well using the above data and this value of σ.

-7

$\cancel{s} = 2.0$ $s^2 = 4$

$df = 29$ CL = 95%

$\chi_R^2 = 45.722$. $\chi_L^2 = 16.047$

95%-CI for σ^2: $\frac{(n-1)\cdot s^2}{\chi_R^2} < \sigma^2 < \frac{(n-1)\cdot s^2}{\chi_L^2}$

$\frac{29 \cdot 4}{45.722} < \sigma^2 < \frac{29 \cdot 4}{16.047}$

$2.53707 < \sigma^2 < 7.22877$

95%-CI for σ: $1.59282 < \sigma < 2.68864$

c. Suppose the Margin of Error needed is 0.1 and the actual standard deviation of the arsenic level of that well is 2.0 ppb, how many more tests data needed to have a confidence-level of 95%?

-3

$E = 0.1$ $s = 2$ CL = 95% $df = 29$ $t_c = 2.045$

$n = \left\lceil \frac{t_c^2 \cdot \cancel{s}}{E^2} \right\rceil = \left\lceil \frac{2.045^2 \cdot 2^2}{0.1^2} \right\rceil = \lceil 1672.81 \rceil = 1673$

$df' = 1672$ $t_c' = 1.9615$

Check. $E = \frac{t_c \cdot s}{\sqrt{n}} = \frac{1.9615 \cdot 2}{\sqrt{1673}} = 0.09591 < 0.1$ ✓

$1673 - 30 = 1643$.

More 1643 test data needed to have a confidence-level of 95%

+11

8. (16:7,9) ThomVax is promoting a vaccine which is supposed to prevent statisticitis, a math disease afflicting students at the East LA College. They are trying to convince the college's health center that the vaccine is worth stocking. ThomVax arranges a 46-week trial with the goal of convincing the center to stock the vaccine, which resulted in an average of 43.5 vaccines per week with a standard deviation of 3.86.

a. Find the 95% confidence interval ~~for~~ the average number of vaccines needed per week.

$n = 46.\ \bar{X} = 43.5 \quad s = 3.86.$

1). Best expect for μ: $\bar{X} = 43.5 \quad df = 45. \quad CL = 95\%. \quad t_c = 2.014$ ✓

2) $E = \dfrac{2.014 \cdot 3.86}{\sqrt{46}} = 1.14622$

3). 95%-CI for μ: $\bar{X} - E < \mu < \bar{X} + E$

$43.5 - 1.14622 < \mu < 43.5 + 1.14622$

$42.35378 < \mu < 44.64622$ ✓

b. The campus health center has agreed to stock and administer the vaccine if the average need is more than 40 vaccines per week. With this sample data, will the health center agree to stock the vaccine? Use $\alpha = 0.05$

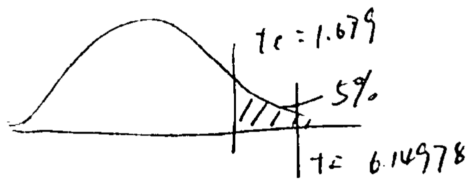
1) Claim that the average need is more than 40 vaccines per week.

2) Hypothesis: $H_0: \mu \le 40.$

$H_1: \mu > 40.$ (claim right-tail)

3) Test-statistic: $t = \dfrac{43.5 - 40}{\frac{3.86}{\sqrt{46}}} = 6.14978$

4). Critical value: $df = 45$, $\alpha = 0.05 \quad t_c = 1.679$



5) Informal conclusion: Reject H_0.

6) Formal Conclusion: At SL 5%, there's enough evidence to ~~the~~ support the claim that the average need is more than 40 vaccines per week

With this sample data, the health center will agree to stock the vaccine ✓

+16

9. (9) The Congressional Budget Office reports that 36% of federal civilian employees have a bachelor's degree. A random sample of 120 employees in the private sector showed that 34 have a bachelor's degree. Does this indicate that the percentage of employees holding bachelor's degrees in the private sector is less than the proportion in the federal civilian sector? Use $\alpha = 0.05$.

1) Claim that the percentage of employees holding bachelor's degrees in the private sector is less than the proportion in the federalcivilian sector. 36%

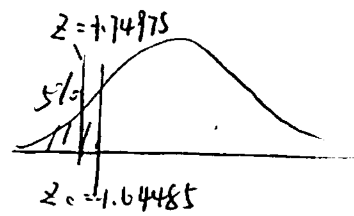
2) Hypothesis: $H_0: p \geq 36\%$

$H_1: p < 36\%$ (claim, left-tail)

$\hat{p} = \frac{x}{n} = \frac{34}{120} = 0.28333$

3) Test-stastic: $z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{0.28333 - 0.36}{\sqrt{\frac{0.36 \cdot 0.64}{120}}} = -1.74975$

4) Critical value: $\alpha = 0.05$ $z_c = \text{Invnorm}(0.05) = -1.64485$



5) Informal Conclusion: Reject H_0.

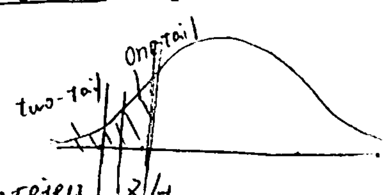
6) Formal Conclusion: At 5%-SL, there's enough evidence to support the claim that the percentage of employees holding bachelor's degrees in the private sector is less than the proportion in the federal civilian sector.

The following problems are conceptual problems, not related to the problem above.

10. (2) For the same data, H_0, and significance level, is it possible that a one-tailed test results in the decision to reject H_0 while a two-tailed test results in the decision to fail to reject H_0? Explain.

1/2

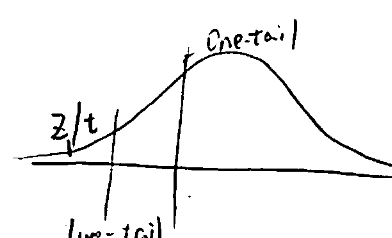
Yes. it's possible.
According to the picture, there's some space between one-tail and two-tail. The value of t/z in it is saying the state that one-tailed to reject H_0, while two-tail to fail to reject.



11. (2) For the same data, H_0, and significance level, if the decision is to reject H_0 based on a two-tailed test, do you also reject H_0 based on a one-tailed test? Explain.

1

Yes. I can also reject H_0 based on a one-tailed test.
Because according to the picture. when the value of test-stastic is in the part of two-tail. it must in one-tail part.



+11.5