Spring 2011 Math 115 Test 3 103.5/100 Name: Natsuko Nishimae 97

Show all necessary work NEATLY, CLEARLY, and UNDERSTANDABLY. Any understatement and/or incorrect statement will be penalized. Answer-by-guessing (i.e., without proper work) will receive no point. There are 100 points available in this test.

1. (1) Write in decimal notation: 7.03×10^{-6}

$= 0.00000703$ ✓

2. (1) Write in scientific notation: 0.00032

$= 3.2\times10^{-4}$ ✓

3. (3) Compute: (write your answer in scientific notation) $(2.6\times10^{-3})(5.3\times10^{-9})$

$= 13.78\times10^{-12}$

$= 1.378\times10^{1}\times10^{-12}$

$= 1.378\times10^{-11}$ ✓

4. (2) Simplify: $-3(4x^3-7x-3)-8(2x^3-3x^2-4)$

$= -12x^3+21x+9-16x^3+24x^2+32$

$= -28x^3+21x+24x^2+41$ ✓

5. (3) Multiply: $(3x-2)(9x^2+6x+4)$

$= 27x^3+18x^2+12x-18x^2-12x-8$

$= 27x^3-8$ ✓

6. (2) Simplify: $(5x-8y)^2$

$= (5x-8y)(5x-8y)$

$= 25x^2-80xy+64y^2$ ✓

7. (2) Factorize: $24x^2-10xy-y^2$

$= (12x+y)(2-y)$ ✓

8. (4) Factorize: $6x^3-14x^2-12x$ Good job!

$= 2x(3x^2-7x-6)$

$= 2x(3x+2)(x-3)$ ✓

9. (3) Factorize: $8x^2+10xy-3y^2$

$= (4x-y)(2x+3y)$ ✓

10. (4) Factorize completely: $2x^3+x^2-18x-9$

$= x^2(2x+1)-9(2x+1)$

$= (2x+1)(x^2-9)$

$= (2x+1)(x+3)(x-3)$ ✓

11. (3) Divide: $\dfrac{6x^3y-12xy+8x}{12x^2y}$

$= \dfrac{6x^3y}{12x^2y}-\dfrac{12xy}{12x^2y}+\dfrac{8x}{12x^2y}$

$= \dfrac{x}{2}-\dfrac{1}{x}+\dfrac{2}{3xy}$ ✓

12. (3) Factorize: $64x^2-112xy+49y^2$

$1 \quad -56=-2 \quad 1$

$8x \quad\quad 7y$

$= (8x-7y)^2$ ✓

x37

66/37.5

13. (3) Factorize: $4x^2-3xy+9y-12x$

$= x(4x-3y)+3(3y-4x)$

$= x(4x-3y)-3(4x-3y)$

$= (4x-3y)(x-3)$ ✓

14. (3) Simplify: $(-4x^4y^2)^3\cdot(-2x^6y^3)^2$

$= (-64x^{12}y^6)\cdot 4x^{12}y^6$

$= -256x^{24}y^{12}$ ✓

15. (4) Simplify: $-x^6y^8(3x^7y^8)^2-x^2(2x^3y^4)^6$

$= -x^6y^8(9x^{14}y^{16})-x^2(64x^{18}y^{24})$

$= -9x^{20}y^{24}-64x^{20}y^{26}$

$= -73x^{20}y^{24}$

16. (4) Simplify: $\dfrac{48x^{-4}y^{-6}z^3}{-84x^{-8}yz^{-3}}$

OK.

$= \dfrac{4x^8z^6}{-7x^4y^7}$ ✓

$= -\dfrac{4x^4z^6}{7y^7}$

17. (4) Simplify: $(-2x^{-3}y^{-2})^{-4}(-3x^{-4}y^2)^{-3}$

$= \left(-\dfrac{2}{x^3y^2}\right)^{-4}\cdot\left(-\dfrac{3y^2}{x^4}\right)^{-3}$

$= \left(-\dfrac{x^3y^2}{2}\right)^4\cdot\left(-\dfrac{x^9}{3y^2}\right)^3$

$= \left(\dfrac{x^{12}y^8}{16}\right)\cdot\left(-\dfrac{x^{12}}{27y^6}\right)$

$= -\dfrac{x^{24}y^8}{432y^6} = \left(-\dfrac{x^{24}y^2}{432}\right)$ ✓

×26

18. (4) Divide: $(2x^2-9x+7)\div(2x+3)$

$$\begin{array}{r} x-6 \\ 2x+3\overline{)\,2x^2-9x+7} \\ -(2x^2+3x) \\ \hline -12x+7 \\ -(-12x-18) \\ \hline 25 \end{array}$$

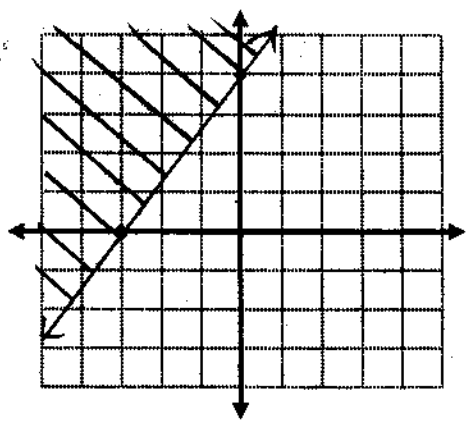
$= x-6+\dfrac{25}{2x+3}$

19. (4) Factorize completely: $16x^4-64y^2$

$= (4x^2)^2-64y^2$

$= (4x^2+8y)(4x^2-8y)$

20. (4) Graph the solution region of $4x-3y\le -12$.



① Boundary

$4x-3y=-12$

x-int: $(-3,0)$ ✓

y-int: $(0,4)$

③ $\le$ → solid ✓

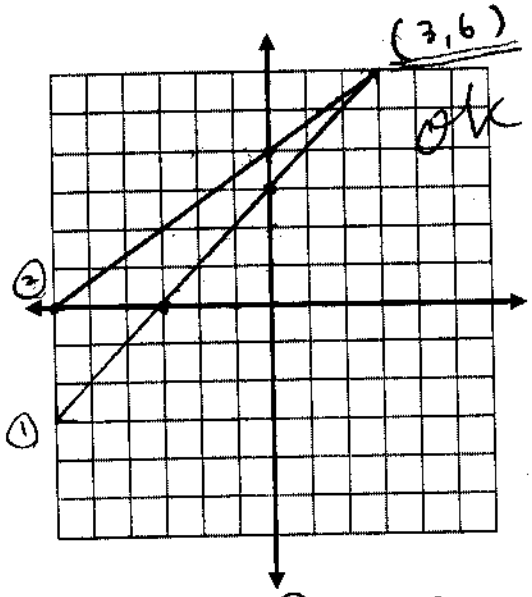
② Test $(0,0)$

$4(0)-3(0)\le -12$

$0\le -12$ ✗ ✓

So, the shade region not containing $(0,0)$

21. (4) Solve by graphing: $\begin{cases} y = x + 3 & ① \\ 2x - 3y = -12 & ② \end{cases}$



① x-int: (-3, 0)
y-int: (0, 3)

② x-int: (-6, 0)
y-int: (0, 4)

22. (4) Solve by addition: $\begin{cases} 3x + 2y = 5 \\ 4x + 5y = 2 \end{cases}$

$\begin{cases} 3x+2y=5 \\ 4x+5y=2 \end{cases}$ $\begin{vmatrix} \times 5 \\ \times(-2) \end{vmatrix}$ $\begin{vmatrix} 15x+10y=25 & ① \\ -8x-10y=-4 & ② \end{vmatrix}$

$7x = 21$

$x = \frac{21}{7}$

$x = 3$ ✓

→ ① $15(3) + 10y = 25$

$45 + 10y = 25$

$10y = 25 - 45$

$10y = -20$

$y = -\frac{20}{10}$

$y = -2$ ✓

So, $(x, y) = (3, -2)$

+20

23. (6) Solve by substitution: $\begin{cases} 3x - y = 2 & ① \\ 5x + 2y = -7 & ② \end{cases}$

① → $3x - y = 2$

$-y = -3x + 2$

$y = 3x - 2$

① → ② $5x + 2y = -7$

$5x + 2(3x - 2) = -7$ ✓

$5x + 6x - 4 = -7$

$11x = -7 + 4$

$11x = -3$

$x = -\frac{3}{11}$

→ ① $y = 3\left(-\frac{3}{11}\right) - 2$

$y = -\frac{9}{11} - \frac{22}{11}$

$y = -\frac{31}{11}$

So, $(x, y) = \left(-\frac{3}{11}, -\frac{31}{11}\right)$ ✓

24. (6) The manager of a store buys a product for $14.50. Find the discount rate (in percent) if it is sold for $12.47.

① $S = 12.47$

$P = 14.50$

$r = ?$

② $S = P(1 - r)$

$12.47 = 14.50(1 - r)$

$12.47 = 14.50 - 14.50r$

$12.47 - 14.50 = -14.50r$

$-2.03 = -14.50r$

$\frac{-2.03}{-14.50} = r$

$0.14 = r$

$r = 0.14 \times 100\%$

$r = 14\%$ ✓

③ So, the discount rate is 14%. ✓

25. (6) A small plane, flying against the wind, flew 360 miles in 3 hours. Flying with the wind, the plane traveled the twice the distance in the same time. Find the rate of the plane.
Note: Do in Four-Step Method.

① Let x be the rate of the plane in calm air and y be the rate of ~~plane~~ wind

② against the wind: $3(x-y)=360$
with the wind: $3(x+y)=360\times 2$

③ $3(x-y)=360 \rightarrow x-y=120$ ①
$3(x+y)=720 \rightarrow x+y=240$ ②

$2x = 360$
$x = \frac{360}{2}$
$x = 180$

→ ② $180+y=240$
$y=240-180$
$y=60$

③ So, the rate of the plane is 60 mph.

26. (8) The total cost of 3 pencils and 2 erasers is $2.74. If the erasers were pencils and the pencils were erasers, the total cost would have been $2.26. Find the price of an eraser.
Note: Do in Four-Step Method.

① Let x be the cost of a pencil and y be the cost of an eraser.

② $\begin{cases} 3x+2y=2.74 \\ 2x+3y=2.26 \end{cases}$

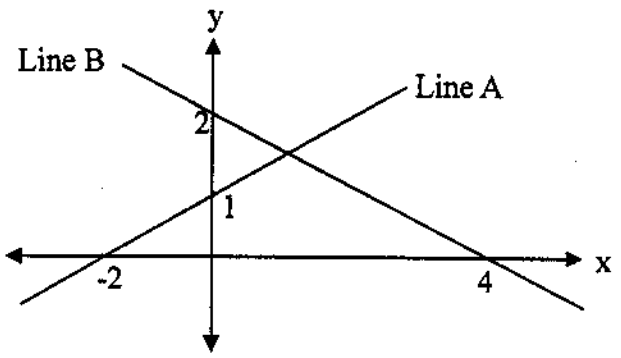
③ $\begin{cases} 3x+2y=2.74 & \times(-2) \\ 2x+3y=2.26 & \times 3 \end{cases} \Rightarrow \begin{cases} -6x-4y=-5.48 \\ 6x+9y=6.78 \end{cases}$

$5y=1.30$
$y=\frac{1.30}{5}$
$y=0.26$

④ So, the price of an eraser is $0.26.

+11.5

27. (7:2,2,3) Consider the following figure. Note that the figure is NOT drawn to scale.



a. Find the equation of Line A. (0,1) (-2,0)

$m_A = \frac{0-1}{-2-0} = \frac{-1}{-2} = \frac{1}{2}$ ✓

Equation → $y = \frac{1}{2}x + 1$

b. Find the equation of Line B. (4,0) (0,2)

$m_B = \frac{2-0}{0-4} = \frac{2}{-4} = -\frac{1}{2}$

Equation → $y = -\frac{1}{2}x + 2$ ✓

c. Find the intersection of those lines.

Intersection → $y_A = y_B$

$\begin{cases} y = \frac{1}{2}x + 1 & (L_A) \\ y = -\frac{1}{2}x + 2 & (L_B) \end{cases}$

$\frac{1}{2}x + 1 = -\frac{1}{2}x + 2$

$\frac{1}{2}x + \frac{1}{2}x = 2 - 1$

$x = 1$ ✓

→ (L_A) $y = \frac{1}{2}x + 1$

$y = \frac{1}{2}(1) + 1$

$y = \frac{1}{2} + \frac{2}{2}$

$y = \frac{3}{2}$ ✓

So, $(x, y) = (1, \frac{3}{2})$

+15

28. (8) Two cars start at the same point and fly in the same direction. The first car, travelling at 60 mph, starts 2 hours earlier than the second car. If the second plane travels at 90 mph, how long does it take the second cars to catch the first car?
Note: Do in Four-Step Method.

① Let x be the time of second car to catch the first car. ✓

②

Type	Time	Rate	Distance
First car	$x+2$	60	$60(x+2)$
Second car	x	90	$90x$
			$60(x+2)=90x$ ✓

③ $60(x+2) = 90x$

$60x + 120 = 90x$

$120 = 90x - 60x$

$120 = 30x$

$\frac{120}{30} = x$

$4 = x$ ✓

④ So, the second car to catch the first car in 4 hours. ✓