Sp20 Math 227 Test 2 Name: SOLUTION

Show your work clearly, neatly, and understandably. Make sure you use proper notation. Also, round the decimal for probability to 5-decimal place and round the percentage to 3-decimal. There are 105 points available.

1. (18:2,3,4,4,5) There are 4 main blood groups: A, B, AB and O, each with positive and negative rhesus. In the United States, the average distribution of blood types is as follows:

O+pos: 38%	B+pos: 9%	A+pos: 34%	AB+pos: 3%
O-neg: 7%	B-neg: 2%	A-neg: 6%	AB-neg: 1%

a. Of 50 people, find the expected number of people have A+pos. $X_A \sim B(50, 0.34)$

$$\mu_A = 50 \cdot 0.34 = 17.$$

b. Of 60 people, find the standard deviation of the number of people have B-neg.

$$X_{B-} \sim B(60, 0.02)$$

$$\sigma^2_{B-} = 60 \cdot 0.02 \cdot 0.98 = 1.176$$

$$\sigma_{B-} = 1.08444$$

c. Find the probability that, of 10 people, at least half of them have AB+pos.

$$X_{AB} \sim B(10, 0.03)$$

$$P(X_{AB} \geq 5) = 1 - P(X_{AB} \leq 4)$$
$$= 1 - \text{binomcdf}(10, 0.03, 4)$$
$$= 0.0000053965 \cong 0.00001$$

d. Find the probability that any of 10 people has A-neg. $X_{A-} \sim B(10, 0.06)$

$$P(X_{A-} \geq 1) = 1 - P(X_{A-} = 0)$$
$$= 1 - \text{binompdf}(10, 0.06, 0)$$
$$= 0.46138$$

e. Of 60 people, find the "usual" interval for the number of people have B-neg.

$$X_{B-} \sim B(60, 0.02)$$

$$\mu_{B-} = 60(0.02) = 1.2$$

$$\sigma_{B-} = 1.08444$$

$$LF = 1.2 - 2(1.08444) = -0.96888$$

$$UF = 1.2 + 2(1.08444) = 3.36888$$

Usual Interval = $(-0.96888, 3.36888)$

2. (6:3,3) HyperBall Lottery draws 4 "regular" numbers from 1 to 50 AND 2 "hyperball" numbers from 1 to 20 numbers. (Just use the notation, do not compute into decimal).

a. Find the probability to correctly select only one hyperball number only (and no correct :regular").

$$\frac{{}_4C_0 \cdot {}_{46}C_4}{{}_{50}C_4} \cdot \frac{{}_2C_1 \cdot {}_{18}C_1}{{}_{20}C_2}$$

1-50	
4C	46W

1-20	
2C	18W

b. Find the probability to correctly select 2 regular numbers and both hyperballs.

$$\frac{{}_4C_2 \cdot {}_{46}C_2}{{}_{50}C_4} \cdot \frac{{}_2C_2 \cdot {}_{18}C_0}{{}_{20}C_2}$$

3. (11:5,2,4) The following lists the probability distribution of X, the number of times a student absent in a twice-a-week class. Extend to find the mean and standard deviation of the number of X.

X	P	px	x^2	px^2
0	0.30	0	0	0
1	0.45	0.45	1	0.45
2	0.22	0.44	4	0.88
3	0.03	0.09	9	0.27
	1	0.98		1.6

$E(X) = 0.98$

$\sigma^2(X) = 1.6 - 0.98^2$

$= 0.6396$

$\sigma(X) = 0.79975$

4. (18:4,3,4,7) A box contains 9 identical marbles (except in colors): 4 green, 3 yellow and 2 red.

a. Three marbles are selected at random, without replacement. Find the probability of selecting any green marble.

$P(G \geq 1) = 1 - P(G=0)$

$= 1 - \dfrac{{}_4C_0 \cdot {}_5C_3}{{}_9C_3}$

$= 1 - \dfrac{1 \cdot 10}{84}$

$= \dfrac{74}{84}$

$= \dfrac{37}{42}$

$= 0.88095$

b. Three marbles are selected at random, one at a time, with replacement. Find the probability of getting 2 yellow marbles first, then a red marble.

$P(Y_1 Y_2 R_3) = \dfrac{3}{9} \cdot \dfrac{3}{9} \cdot \dfrac{2}{9}$

$= \dfrac{2}{81} = 0.02469$

c. Three marbles are selected at random, with replacement. Find the probability of getting 1 red marble, regardless the order.

$R \sim B(3, \frac{2}{9})$

$P(R=1) = \text{binompdf}(3, \frac{2}{9}, 1)$

$= 0.40329$

d. Construct the probability distribution for the number of yellow marbles selected in a procedure of selecting three marbles without replacement.

$Y = \{0, 1, 2, 3\}$

$P(Y=0) = \dfrac{{}_3C_0 \, {}_6C_3}{{}_9C_3} = \dfrac{1 \cdot 20}{84} = \dfrac{20}{84}$

$P(Y=1) = \dfrac{{}_3C_1 \cdot {}_6C_2}{{}_9C_3} = \dfrac{3 \cdot 15}{84} = \dfrac{45}{84}$

$P(Y=2) = \dfrac{{}_3C_2 \cdot {}_6C_1}{{}_9C_3} = \dfrac{3 \cdot 6}{84} = \dfrac{18}{84}$

$P(Y=3) = \dfrac{{}_3C_3 \cdot {}_6C_0}{{}_9C_3} = \dfrac{1 \cdot 1}{84} = \dfrac{1}{84}$

Y	P
0	$\frac{20}{84}$
1	$\frac{45}{84}$
2	$\frac{18}{84}$
3	$\frac{1}{84}$
	$\frac{84}{84} = 1$

5. (14:3,3,2,3,3) Let Let $P(A)=0.7, P(A\mid B)=0.5$ and $P(B\mid A)=0.4$.

a. $P(A\cap B) = P(A)\cdot P(B|A)$

$= 0.7 \times 0.4$

$= \underline{\underline{0.28}}$

b. $P(B) = \underline{\underline{0.56}}$

$P(A|B) = \dfrac{P(A\cap B)}{P(B)} \Rightarrow P(B) = \dfrac{P(A\cap B)}{P(A|B)}$

$= \dfrac{0.28}{0.5}$

$= 0.56$

c. $P(A\cup B)$

$= P(A) + P(B) - P(A\cap B)$

$= 0.7 + 0.56 - 0.28 = \underline{\underline{0.98}}$

d. $P(\overline{A}\cap B)$

$= P(B) - P(A\cap B)$

$= 0.56 - 0.28$

$= \underline{\underline{0.28}}$

e. $P(B\mid \overline{A}) = \dfrac{P(\overline{A}\cap B)}{P(\overline{A})}$

$= \dfrac{0.28}{1-0.7}$

$= \dfrac{0.28}{0.3}$

$= \underline{\underline{0.93333}}$

6. (20:9,4,7) A game of "Tri-Tins" draws three cans of soda, without replacement, from a box containing 12 identical cans (except in brand): 5 Sprite, 4 Coke and 3 Fanta. The player wins \$10 if all cans selected are Sprite, \$30 if all are Coke and \$100 if all are Fanta. The player losses \$1 otherwise. Construct the probability distribution of the payout. Then extend to compute the expectation and standard deviation of the payout.

Note: DO IN FRACTION first.

Let X be the payout

$X = \{10, 30, 100, -1\}$.

$P(X=10) = P(3\text{ sprites})$

$= \dfrac{{}_5C_3 \cdot {}_7C_0}{{}_{12}C_3} = \dfrac{10\cdot 1}{220} = \dfrac{10}{220}$

$P(X=30) = P(3\text{ cokes})$

$= \dfrac{{}_4C_3 \cdot {}_8C_0}{{}_{12}C_3} = \dfrac{4\cdot 1}{220} = \dfrac{4}{220}$

$P(X=100) = P(3\text{ Fantas})$

$= \dfrac{{}_3C_3 \cdot {}_9C_0}{{}_{12}C_3} = \dfrac{1\cdot 1}{220} = \dfrac{1}{220}$

$P(X=-1) = 1 - \left(\dfrac{10+4+1}{220}\right) = \dfrac{205}{220}$

X	P	PX	X^2	PX^2
10	$\frac{10}{220}$	$\frac{100}{220}$	100	$\frac{1000}{220}$
30	$\frac{4}{220}$	$\frac{120}{220}$	900	$\frac{3600}{220}$
100	$\frac{1}{220}$	$\frac{100}{220}$	10000	$\frac{10000}{220}$
-1	$\frac{205}{220}$	$-\frac{205}{220}$	1	$\frac{205}{220}$
	1	$\frac{115}{220}$		$\frac{14805}{220}$

$E(X) = \dfrac{115}{220} = \dfrac{23}{44} = \underline{\underline{0.52273}}$

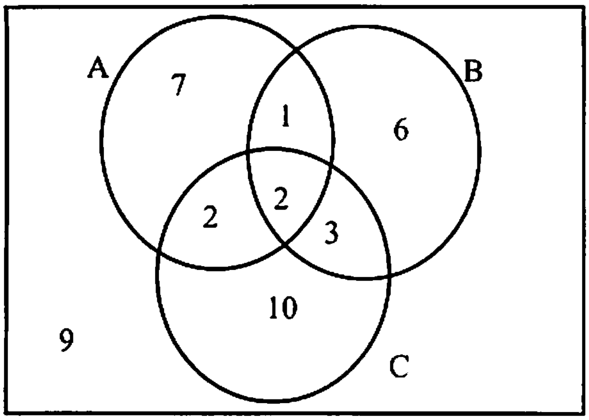
$Var(X) = \dfrac{14805}{220} - \left(\dfrac{23}{44}\right)^2$

$= 67.02221$

$\sigma(X) = \underline{\underline{8.18671}}$

7. (9:3,3,3) The following Venn Diagram describes the number of elements in A , B, and C.

(Just use the notation, do not compute into decimal).



Suppose <u>four</u> elements are randomly selected, <u>without replacement</u>, from the diagram, find the probability that:

a. At least one of them from A.

$$P(A \geq 1) = 1 - P(A=0)$$
$$= 1 - \frac{{}_{12}C_0 \cdot {}_{28}C_4}{{}_{40}C_4}$$
or $$= 1 - \frac{28}{40} \cdot \frac{27}{39} \cdot \frac{26}{38} \cdot \frac{25}{37}$$

b. Only one of them from B.

$$P(B=1) = \frac{{}_{12}C_1 \cdot {}_{28}C_3}{{}_{40}C_4}$$

c. The first two are from C and the rest is not from C.

$$P(C C \bar{C} \bar{C}) = \frac{17}{40} \cdot \frac{16}{39} \cdot \frac{23}{38} \cdot \frac{22}{37}$$

or $$= \frac{{}_{17}P_2 \cdot {}_{23}P_2}{{}_{40}P_4}$$

8. (9:2,3,4) At a grocery store, the number of customers arriving at the checkout line follows Poisson distribution with a rate of 1.7 customers per 5-minute. Find the probability that

Note: Write the calculator notation before giving the calculator result.

a. At most 4 arrivals in a 5-minute period.

Let X be # of customers in 5 min

$X \sim$ Poisson (1.7)

$P(X \leq 4) = \text{poissoncdf}(1.7, 4) = \underline{0.97039}$

b. At least 20 arrivals in a 5-minute period.

$$P(X \geq 20) = 1 - P(X \leq 19)$$
$$= 1 - \text{poissoncdf}(1.7, 19)$$
$$= \underline{0}$$

c. No arrival in the next 2-minute period.

$P(X=0$, in 2 min$)$

$$= \text{poissonpdf}\left(1.7 \times \tfrac{2}{5}, 0\right)$$
$$= \underline{0.50662}$$