

Show all necessary work NEATLY, CLEARLY, and SYSTEMATICALLY for full point. 51-point

1. (6: 2-points each row) Write the values of $\sec \theta$, $\sin \theta$, and $\cot \theta$ for the angles 0° , 30° , 45° , 60° , and 90° .

θ	0°	30°	45°	60°	90°
$\sin \theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\sec \theta$	1	$\frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2	UD
$\cot \theta$	UD	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	0

2. (8) Let $\sec \theta = x, x \in Q_3$. Find the values of the other 5 trigonometric functions of θ in terms of x . Don't need to rationalize the denominators.

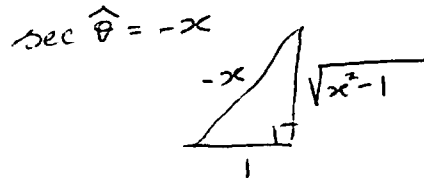
$$\cos \theta = \frac{1}{x}$$

$$\csc \theta = \frac{x}{\sqrt{x^2-1}}$$

$$\sin \theta = \frac{\sqrt{x^2-1}}{x}$$

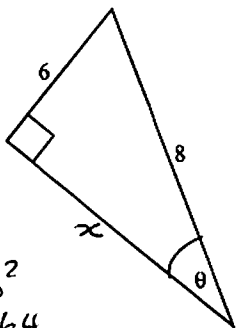
$$\tan \theta = \sqrt{x^2-1}$$

$$\cot \theta = \frac{1}{\sqrt{x^2-1}}$$



3. (9: 1.5 each) Consider the following right triangle:

Find: (You need to rationalize the denominators)



$$\begin{aligned} x^2 + 6^2 &= 8^2 \\ x^2 + 36 &= 64 \\ x^2 &= 28 \\ x &= 2\sqrt{7} \end{aligned}$$

a. $\sin \theta = \frac{6}{8} = \frac{3}{4}$

d. $\sec \theta = \frac{4}{\sqrt{7}} = \frac{4\sqrt{7}}{7}$

b. $\cos \theta = \frac{2\sqrt{7}}{8} = \frac{\sqrt{7}}{4}$

e. $\csc \theta = \frac{4}{3}$

c. $\tan \theta = \frac{6}{2\sqrt{7}} = \frac{3}{\sqrt{7}} = \frac{3\sqrt{7}}{7}$

f. $\cot \theta = \frac{\sqrt{7}}{3}$

4. (6: 1 each) Find six trigonometric functions of θ if $(-6, -8)$ is on the terminal side of θ , in standard position.

$$\sin \theta = \frac{-8}{10} = -0.8$$

$$\cos \theta = -\frac{6}{10} = -0.6$$

$$\tan \theta = \frac{-8}{-6} = \frac{4}{3} = 1.33333$$

$$\sec \theta = -\frac{10}{6} = -\frac{5}{3} = -1.66667 \quad \csc \theta = -\frac{10}{8} = -\frac{5}{4} = -1.25 \quad \cot \theta = \frac{-6}{-8} = \frac{3}{4} = 0.75$$

$$\begin{aligned} x^2 + y^2 &= r^2 \\ (-6)^2 + (-8)^2 &= r^2 \\ 36 + 64 &= r^2 \\ r^2 &= 100 \\ r &= 10 \end{aligned}$$

5. (7) Prove: $\tan x + \cot x = \sec x \cdot \csc x$

$$\text{Pf: } (\Rightarrow) \text{ LHS} = \tan x + \cot x$$

$$= \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$$

$$= \frac{\sin^2 x}{\cos x \sin x} + \frac{\cos^2 x}{\sin x \cos x}$$

$$= \frac{\sin^2 x + \cos^2 x}{\cos x \sin x}$$

$$= \frac{1}{\cos x \sin x}$$

$$= \frac{1}{\cos x} \cdot \frac{1}{\sin x}$$

$$= \sec x \cdot \csc x$$

$$= \text{RHS} \quad \square$$

6. (7) Prove: $\frac{\cot A + \cot B}{\tan A + \tan B} = \cot A \cot B$

$$\text{Pf: } (\Rightarrow) \text{ LHS} = \frac{\cot A + \cot B}{\tan A + \tan B}$$

$$= \frac{\cot A + \cot B}{\frac{1}{\cot A} + \frac{1}{\cot B}} \cdot \frac{\cot A \cot B}{\cot A \cot B}$$

$$= \frac{(\cot A + \cot B) \cot A \cot B}{\cot B + \cot A}$$

$$= \cot A \cdot \cot B$$

$$= \text{RHS} \quad \square$$

7. (8:2 each) Write each of the following in terms of $\sin x$ only:

a. $\cos x = \pm \sqrt{1 - \sin^2 x}$

$$\sin^2 x + \cos^2 x = 1$$

$$\cos^2 x = 1 - \sin^2 x$$

$$\cos x = \pm \sqrt{1 - \sin^2 x}$$

b. $\csc x$

$$= \frac{1}{\sin x}$$

c. $\tan x = \pm \frac{\sin x}{\sqrt{1 - \sin^2 x}}$

d. $\sec x = \pm \frac{1}{\sqrt{1 - \sin^2 x}}$

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8. (4:2.2) Use calculator to compute:

$$\csc 264^\circ = \frac{1}{\sin 264^\circ} = \underline{\underline{-1.00551}}$$

$$\cot 173^\circ = \frac{1}{\tan 173^\circ} = \underline{\underline{-8.14435}}$$

9. (4:2.2) Suppose $\theta \in Q_1$, use calculator to find the θ such that:

$$\cot \theta = 7.92$$

$$\tan \theta = \frac{1}{7.92}$$

$$\theta = \tan^{-1}\left(\frac{1}{7.92}\right)$$

$$\theta = \underline{\underline{7.19624^\circ}}$$

$$\sec \theta = 1.74$$

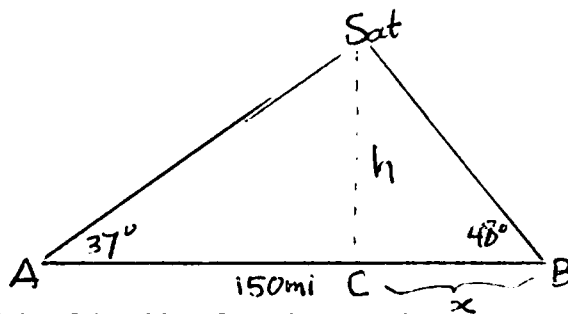
$$\cos \theta = \frac{1}{1.74}$$

$$\theta = \cos^{-1}\left(\frac{1}{1.74}\right)$$

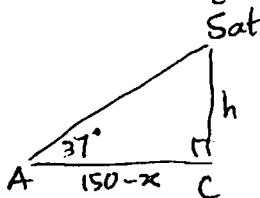
$$\theta = \underline{\underline{54.92049^\circ}}$$

10. (10:2.8) Radar A and Radar B are located 150 miles from each other. They are both tracking an object in the sky at 37° and 48° angles of elevation, respectively, and located above the ground between them.

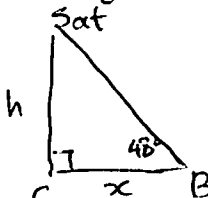
- a. Sketch the situation. Make sure you put all the given information in the sketch properly.



- b. Find the height of the object from the ground.



$$\tan 37^\circ = \frac{h}{150-x}$$



$$\tan 48^\circ = \frac{h}{x}$$

$$\rightarrow h = (150-x) \tan 37^\circ \quad (1) \qquad h = x \tan 48^\circ \quad (2)$$

$$\text{So, } (1) = (2): \quad x \tan 48^\circ = (150-x) \tan 37^\circ$$

$$x \tan 48^\circ = 150 \tan 37^\circ - x \tan 37^\circ$$

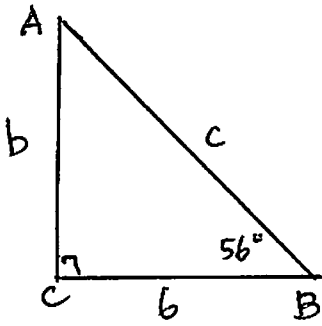
$$x \tan 48^\circ + x \tan 37^\circ = 150 \tan 37^\circ$$

$$x (\tan 48^\circ + \tan 37^\circ) = 150 \tan 37^\circ$$

$$x = \frac{150 \tan 37^\circ}{\tan 48^\circ + \tan 37^\circ}$$

$$(2): h = x \tan 48^\circ = \frac{150 \tan 37^\circ \tan 48^\circ}{\tan 48^\circ + \tan 37^\circ} = \underline{\underline{67.34161 \text{ mi}}}$$

11. (8:1,4,3) Let $\triangle ABC$ have a right angle at C. Let $B = 56^\circ$ and $a = 6$. Solve the triangle. (i.e., find A, b, and c).



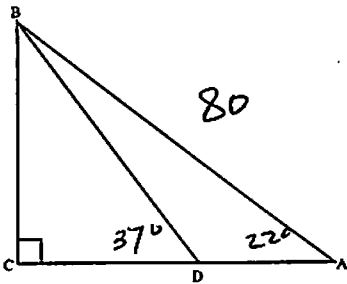
A : $A + B = 90^\circ$
 $A = 90^\circ - B$
 $A = 90^\circ - 56^\circ$
 $A = 34^\circ$

b : $\tan 56^\circ = \frac{b}{6}$
 $6 \tan 56^\circ = b$
 $8.89537 = b$

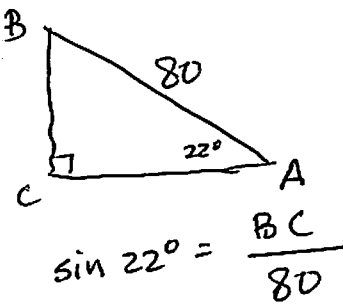
c $\cos 56^\circ = \frac{6}{c}$
 $c \cos 56^\circ = 6$
 $c = \frac{6}{\cos 56^\circ}$

$c = 10.72975$

12. (8:2 each) Let $A = 22^\circ$, $\angle BDC = 37^\circ$, $AB = 80$. Find BC, AC, CD and AD. Organize your work!



BC :



$\sin 22^\circ = \frac{BC}{80}$

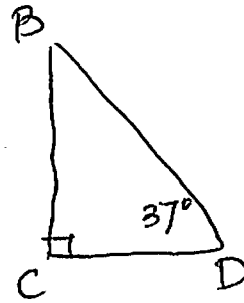
$80 \sin 22^\circ = BC$

$BC = 29.96853$

AC $\cos 22^\circ = \frac{AC}{80}$

$80 \cos 22^\circ = AC$

$AC = 74.17471$



CD : $\tan 37^\circ = \frac{BC}{CD}$

$\tan 37^\circ = \frac{29.96853}{CD}$

$CD \tan 37^\circ = 29.96853$

$CD = \frac{29.96853}{\tan 37^\circ}$

$CD = 39.76958$

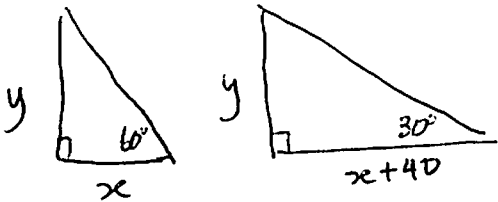
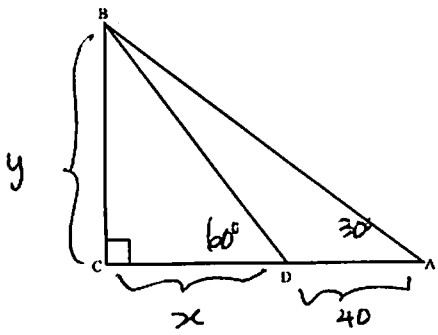
AD

$AD + CD = AC$

$AD + 39.76958 = 74.17471$

$AD = 34.40513$

13. (8:6.2) Let $A = 30^\circ$, $\angle BDC = 60^\circ$, $AD = 40$. Find CD and then BC . Organize your work!



Since they are special angles

Then:

$$(1) y = x\sqrt{3}$$

$$(2) x + 40 = y\sqrt{3}$$

$$\Rightarrow x + 40 = (x\sqrt{3})(\sqrt{3})$$

$$x + 40 = 3x$$

$$40 = 2x$$

$$20 = x$$

$$CD = \underline{20}$$

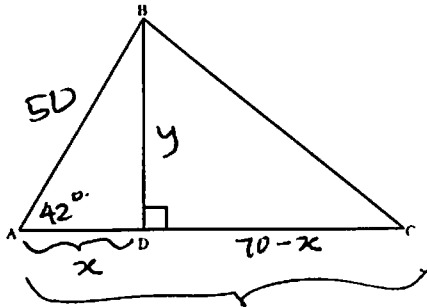
$$y = x\sqrt{3}$$

$$y = 20\sqrt{3}$$

$$BC = \underline{20\sqrt{3}}$$

14. (10:7.3) In $\triangle ABC$, $AB = 50$ and $AC = 70$ and $A = 42^\circ$. Find $\angle C$ and $\angle ABC$.

You may need to explain your work in steps.



$$\cos 42^\circ = \frac{x}{50}$$

$$50 \cos 42^\circ = x$$

$$x = 37.15724$$

$$AD = 37.15724$$

$$CD = 70 - x$$

$$CD = 70 - 37.15724$$

$$CD = 32.84276$$

$$\sin 42^\circ = \frac{y}{50}$$

$$50 \sin 42^\circ = y$$

$$y = 33.45653$$

$$BD = 33.45653$$

$$\tan C = \frac{BD}{CD}$$

$$\tan C = \frac{33.45653}{32.84276} = 1.01869$$

$$C = \tan^{-1}(1.01869)$$

$$C = \underline{45.53040^\circ}$$

$$\angle ABC + A + C = 180^\circ$$

$$\angle ABC = 180^\circ - A - C$$

$$\angle ABC = 180^\circ - 42^\circ - 45.53040^\circ$$

$$\angle ABC = \underline{92.46960^\circ}$$