Fall 2019 Math 241 Test 2 Non-Calculator Name: SOLUTION

Show all necessary work NEATLY, CLEARLY, and SYSTEMATICALLY for full point.
Leave your answer in $\pi$ if it involves any. Don't change to decimal unless told to do so.

1. (18: 3 each) Compute and write the EXACT VALUES. Make sure you show logical steps leading to the answers.

a. $\sin(-390°) = -\sin 390° = -\sin 30° = -\frac{1}{2}$

b. $\cos(-480°) = \cos 480° = \cos 120° = -\cos 60° = -\frac{1}{2}$

c. $\tan\left(\frac{17\pi}{6}\right) = \tan\left(\frac{5\pi}{6}\right) = -\tan\frac{\pi}{6} = -\frac{\sqrt{3}}{3}$

d. $\csc\left(-\frac{7\pi}{6}\right) = \frac{1}{\sin\left(-\frac{7\pi}{6}\right)} = -\frac{1}{\sin\left(\frac{7\pi}{6}\right)} = -\frac{1}{-\sin\left(\frac{\pi}{6}\right)} = \frac{1}{\sin\left(\frac{\pi}{6}\right)} = \frac{1}{1/2} = 2$

e. $\cot(705°) = \frac{1}{\tan 705°} = \frac{1}{\tan 345°} = \frac{1}{-\tan 15°}$

(can't go on w/o calculator yet)

f. $\sec\left(-\frac{5\pi}{3}\right) = \frac{1}{\cos\left(-\frac{5\pi}{3}\right)} = \frac{1}{\cos\left(\frac{5\pi}{3}\right)} = \frac{1}{\cos\frac{\pi}{3}} = \frac{1}{1/2} = 2$

2. (5:3,2) Let a sector be with central angle $\theta = 200°$ and radius 9 ft.

a. Find the perimeter of the sector

$\theta = 200° \cdot \frac{\pi}{180°} = \frac{10\pi}{9}$

$P = 2r + \theta r = 2 \cdot 9 + \frac{10\pi}{9} \cdot 9$

$= (18 + 10\pi)$ ft

b. Find the area of the sector.

$A = \frac{1}{2}\theta r^2$

$= \frac{1}{2} \cdot \frac{10\pi}{9} \cdot 9^2$

$= 45\pi$ ft$^2$

3. (6) A tire rolls with at the rate of 120 rpm on a circle of radius 55cm. Find the distance traveled by the tire if it moves for 2 hours.

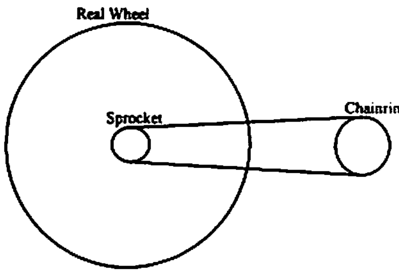
$\omega = 120$ rpm $= 120 \cdot 2\pi$ rad/min $= 120 \cdot 2\pi \cdot 60$ rad/hr $= 14400\pi$ rad/hr

$v = 14400\pi$ rad/hr $\cdot$ 55 cm $= 7920\pi$ m/hr

$s = vt = 7920\pi$ m/hr $\cdot$ 2hr $= 15{,}840\pi$ m

$= 15.84\pi$ km

4. (10:2,2,1,2,1,2) Consider the circles in a bicycle. Suppose the radius of the chainring is 7 cm, of the sprocket 4 cm, and of the rear wheel 50 cm. Let the cyclist is pedaling at 60 rpm. Through the following set-up, we will be able to find the speed of the bicycle.




60 rpm $= 120\pi$ rad/min

a. Find the angular velocity of the chainring.

$\omega_c = 60 \text{ rpm} = 60 \cdot 2\pi \text{ rad/min}$
$= 120\pi \text{ rad/min}$

b. Find the linear velocity of the chainring.

$v_c = 120\pi \cdot 7 \text{ cm/min}$
$= 840\pi \text{ cm/min}$

c. Find the linear velocity of the sprocket.

$v_s = v_c = 840\pi \text{ cm/min}$

d. Find the angular velocity of the sprocket.

$\omega_s = \dfrac{v_s}{r} = \dfrac{840\pi \text{ cm/min}}{4 \text{ cm}}$
$= 210\pi \text{ rad/min}$

e. Find the angular velocity of the rear wheel.

$\omega_r = \omega_s = 210\pi \text{ rad/min}$

f. Find the linear velocity of the rear wheel. This is the speed of the bicycle.

$v_r = 210\pi \cdot 50 \text{ cm/min}$
$= 10{,}500\pi \text{ cm/min}$
$= 105\pi \text{ m/min}$
$= 105\pi \cdot 60 \text{ m/hr}$
$= 6300\pi \text{ m/hr} = 6.3\pi \text{ km/hr}$

5. (13: 1,1,2,2,2,2,3) Let $y = 3 - 4\sin\left(3x + \frac{\pi}{2}\right) = 3 - 4\sin 3\left(x + \frac{\pi}{6}\right)$

a. Center $= 3$

b. Amplitude $= 4$

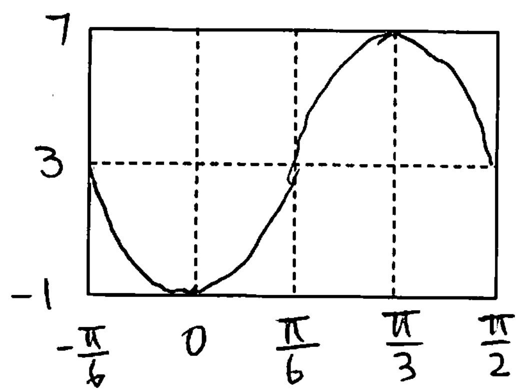
c. Range $= [-1, 7]$

d. Period $= \frac{2\pi}{3}$

e. phase-shift $= -\frac{\pi}{6}$

f. y-intercept $(0, -1)$.
$3 - 4\sin\left(\frac{\pi}{2}\right) = 3 - 4$
$= -1$

g. Sketch the graph of one period in the provided window. Make sure you provide the details.



6. (13: 1,1,2,2,2,2,3) Let $y=-3+3\cos\left(\pi x+\frac{\pi}{2}\right)$ $= -3 + 3\cos\pi\left(x+\frac{1}{2}\right)$

a. Center $= -3$

b. Amplitude $= 3$

c. Range $= [-6, 0]$

d. Period $= \frac{2\pi}{\pi} = 2$

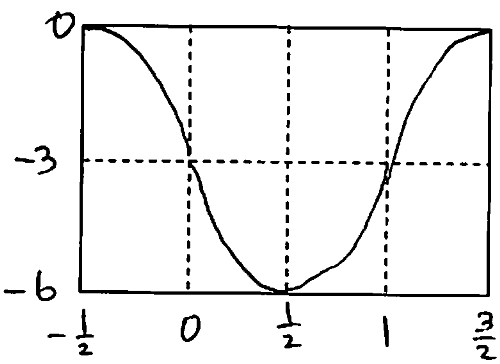
e. phase-shift $= -\frac{1}{2}$

f. y-intercept $(0,-3)$

$-3+3\cos\left(\frac{\pi}{2}\right)$

$= -3 + 3(0) = -3$

g. Sketch the graph of one period in the provided window. Make sure you provide the details.




7. (10: 2,2,3,3) Compute:

a. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$= -\frac{\pi}{3}$

b. $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) = -\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$

$= -\frac{\pi}{6}$

c. $\cos^{-1}(\cos 240°) = \cos^{-1}\left(-\cos 60°\right)$

$= \cos^{-1}\left(-\frac{1}{2}\right)$

$= \pi - \cos^{-1}\left(\frac{1}{2}\right)$

$= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

d. $\sin^{-1}(\sin 210°)$

$= \sin^{-1}\left(-\sin 30°\right)$

$= \sin^{-1}\left(-\frac{1}{2}\right) = -\sin^{-1}\left(\frac{1}{2}\right)$

$= -\frac{\pi}{6}$

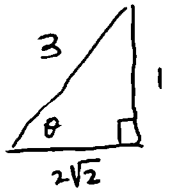
8. (8: 4,4) Compute:

a. $\cot\left(\csc^{-1} 3\right)$

Let $\theta = \csc^{-1} 3$

$\csc\theta = 3$

$\cot\theta = 2\sqrt{2}$.

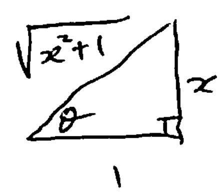



So, $\cot\left(\csc^{-1}3\right) = 2\sqrt{2}$

b. $\sec\left(\tan^{-1} x\right)$. Note: Assume $x > 0$

Let $\theta = \tan^{-1} x$

$\tan\theta = x$




$\sec\theta = \sqrt{x^2+1}$

So, $\sec\left(\tan^{-1}x\right) = \sqrt{x^2+1}$

9. (6) Prove: $\cot^2 x - \cos^2 x = \cos^2 x \cdot \cot^2 x$

Pf ($\Longrightarrow$) LHS $= \cot^2 x - \cos^2 x$

$$= \frac{\cos^2 x}{\sin^2 x} - \cos^2 x$$

$$= \frac{\cos^2 x}{\sin^2 x} - \frac{\cos^2 x \sin^2 x}{\sin^2 x}$$

$$= \frac{\cos^2 x - \cos^2 x \sin^2 x}{\sin^2 x}$$

$$= \frac{\cos^2 x (1 - \sin^2 x)}{\sin^2 x}$$

$$= \frac{\cos^2 x}{\phantom{x}} \cdot \frac{\cos^2 x}{\sin^2 x}$$

$$= \cos^2 x \cdot \cot^2 x$$

$$= \text{RHS}$$

10. (7) Prove: $\dfrac{\sin x + \tan x}{\cot x + \csc x} = \sin x \cdot \tan x$

Pf ($\Longrightarrow$) LHS $= \dfrac{\sin x + \tan x}{\cot x + \csc x}$

$$= \frac{\sin x + \frac{\sin x}{\cos x}}{\frac{\cos x}{\sin x} + \frac{1}{\sin x}} \cdot \frac{\sin x \cos x}{\sin x \cos x}$$

$$= \frac{\sin^2 x \cos x + \sin^2 x}{\cos^2 x + \cos x}$$

$$= \frac{\sin^2 x (\cos x + 1)}{\cos x (\cos x + 1)}$$

$$= \frac{\sin^2 x}{\cos x}$$

$$= \sin x \cdot \frac{\sin x}{\cos x}$$

$$= \sin x \cdot \tan x$$

$$= \text{RHS}$$

11. (8) Prove: $\dfrac{\cos x}{1 - \sin x} - \dfrac{\cos x}{1 + \sin x} = 2 \tan x$

Pf: ($\Longrightarrow$) LHS $= \dfrac{\cos x}{1 - \sin x} - \dfrac{\cos x}{1 + \sin x}$

$$= \frac{\cos x (1 + \sin x)}{(1 + \sin x)(1 - \sin x)} - \frac{\cos x (1 - \sin x)}{(1 + \sin x)(1 - \sin x)}$$

$$= \frac{\cos x + \cos x \sin x - \cos x + \cos x \sin x}{1 - \sin^2 x}$$

$$= \frac{2 \cos x \sin x}{\cos^2 x}$$

$$= \frac{2 \sin x}{\cos x}$$

$$= 2 \tan x$$

$$= \text{RHS}$$