

Show all necessary work NEATLY, CLEARLY, and SYSTEMATICALLY for full point. Leave your answer in π if it involves any. Don't change to decimal unless told to do so.

1. (18: 3 each) Compute and write the EXACT VALUES. Make sure you show logical steps leading to the answers.

a. $\sin(-390^\circ) = -\sin 390^\circ$
 $= -\sin 30^\circ$
 $= -\frac{1}{2}$

b. $\cos(-480^\circ) = \cos 480^\circ$
 $= \cos 120^\circ$
 $= -\cos 60^\circ$
 $= -\frac{1}{2}$

c. $\tan\left(\frac{17\pi}{6}\right) = \tan\left(\frac{5\pi}{6}\right)$
 $= -\tan \frac{\pi}{6}$
 $= -\frac{\sqrt{3}}{3}$

d. $\csc\left(-\frac{7\pi}{6}\right) = \frac{1}{\sin\left(-\frac{7\pi}{6}\right)} = -\frac{1}{\sin\left(\frac{7\pi}{6}\right)}$
 $= -\frac{1}{-\sin\left(\frac{\pi}{6}\right)} = \frac{1}{\sin\left(\frac{\pi}{6}\right)}$

e. $\cot(705^\circ) = \frac{1}{\tan 705^\circ} = \frac{1}{\tan 345^\circ} = \frac{1}{-\tan 15^\circ}$
 (can't go on w/o calculator yet)

f. $\sec\left(-\frac{5\pi}{3}\right) = \frac{1}{\cos\left(-\frac{5\pi}{3}\right)}$
 $= \frac{1}{\cos\left(\frac{5\pi}{3}\right)} = \frac{1}{\cos\frac{\pi}{3}} = \frac{1}{\frac{1}{2}} = 2$

2. (5:3.2) Let a sector be with central angle $\theta = 200^\circ$ and radius 9 ft.

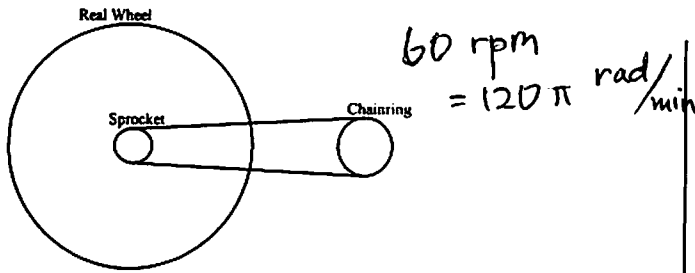
a. Find the perimeter of the sector
 $\theta = 200^\circ \cdot \frac{\pi}{180^\circ} = \frac{10\pi}{9}$
 $P = 2r + \theta r = 2 \cdot 9 + \frac{10\pi}{9} \cdot 9$
 $= (18 + 10\pi) \text{ ft}$

b. Find the area of the sector.
 $A = \frac{1}{2} \theta r^2$
 $= \frac{1}{2} \cdot \frac{10\pi}{9} \cdot 9^2$
 $= 45\pi \text{ ft}^2$

3. (6) A tire rolls with at the rate of 120 rpm on a circle of radius 55cm. Find the distance traveled by the tire if it moves for 2 hours.

$\omega = 120 \text{ rpm} = 120 \cdot 2\pi \text{ rad/min} = 120 \cdot 2\pi \cdot 60 \text{ rad/hr} = 14400\pi \text{ rad/hr}$
 $v = 14400\pi \text{ rad/hr} \cdot 55 \text{ cm} = 7920\pi \text{ m/hr}$
 $s = vt = 7920\pi \text{ m/hr} \cdot 2 \text{ hr} = 15,840\pi \text{ m}$
 $= 15.84\pi \text{ km}$

4. (10:2,2,1,2,1,2) Consider the circles in a bicycle. Suppose the radius of the chainring is 7 cm, of the sprocket 4 cm, and of the rear wheel 50 cm. Let the cyclist is pedaling at 60 rpm. Through the following set-up, we will be able to find the speed of the bicycle.



- a. Find the angular velocity of the chainring.

$$\omega_c = 60 \text{ rpm} = 60 \cdot 2\pi \text{ rad/min} = 120\pi \text{ rad/min}$$

- b. Find the linear velocity of the chainring.

$$v_c = 120\pi \cdot 7 \text{ cm/min} = 840\pi \text{ cm/min}$$

- c. Find the linear velocity of the sprocket.

$$v_s = v_c = 840\pi \text{ cm/min}$$

- d. Find the angular velocity of the sprocket.

$$\omega_s = \frac{v_s}{r} = \frac{840\pi \text{ cm/min}}{4 \text{ cm}} = 210\pi \text{ rad/min}$$

- e. Find the angular velocity of the rear wheel.

$$\omega_r = \omega_s = 210\pi \text{ rad/min}$$

- f. Find the linear velocity of the rear wheel. This is the speed of the bicycle.

$$v_r = 210\pi \cdot 50 \text{ cm/min} = 10,500\pi \text{ cm/min}$$

$$= 105\pi \text{ m/min}$$

$$= 105\pi \cdot 60 \text{ m/hr}$$

$$= 6300\pi \text{ m/hr} = 6.3\pi \text{ km/hr}$$

5. (13:1,1,2,2,2,2,3) Let $y = 3 - 4\sin\left(3x + \frac{\pi}{2}\right) = 3 - 4\sin 3\left(x + \frac{\pi}{6}\right)$

a. Center = 3

c. Range = $[-1, 7]$

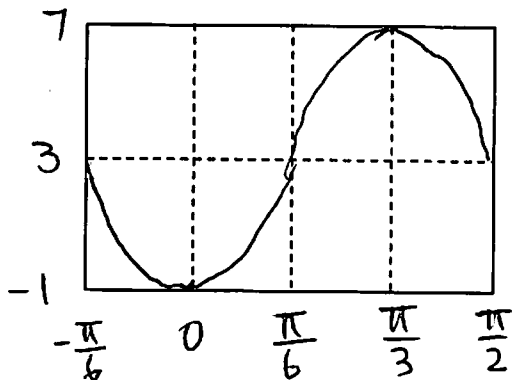
e. phase-shift = $-\frac{\pi}{6}$

b. Amplitude = 4

d. Period = $\frac{2\pi}{3}$

f. y-intercept $(0, -1)$.
 $3 - 4\sin\left(\frac{\pi}{2}\right) = 3 - 4 = -1$

- g. Sketch the graph of one period in the provided window. Make sure you provide the details.



6. (13: 1,1,2,2,2,2,3) Let $y = -3 + 3 \cos\left(\pi x + \frac{\pi}{2}\right) = -3 + 3 \cos \pi\left(x + \frac{1}{2}\right)$

a. Center = -3

c. Range = $[-6, 0]$

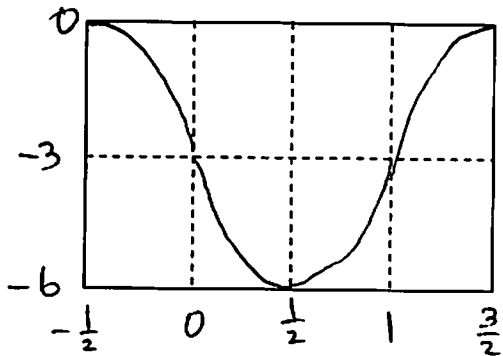
e. phase-shift = $-\frac{1}{2}$

b. Amplitude = 3

d. Period = $\frac{2\pi}{\pi} = 2$

f. y-intercept $(0, -3)$
 $-3 + 3 \cos\left(\frac{\pi}{2}\right)$
 $= -3 + 3(0) = -3$

g. Sketch the graph of one period in the provided window. Make sure you provide the details.



7. (10: 2,2,3,3) Compute:

a. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$
 $= -\frac{\pi}{3}$

b. $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) = -\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$
 $= -\frac{\pi}{6}$

c. $\cos^{-1}(\cos 240^\circ) = \cos^{-1}(-\cos 60^\circ)$
 $= \cos^{-1}\left(-\frac{1}{2}\right)$
 $= \pi - \cos^{-1}\left(\frac{1}{2}\right)$
 $= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

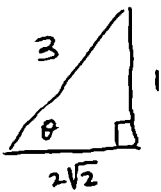
d. $\sin^{-1}(\sin 210^\circ)$
 $= \sin^{-1}(-\sin 30^\circ)$
 $= \sin^{-1}\left(-\frac{1}{2}\right) = -\sin^{-1}\left(\frac{1}{2}\right)$
 $= -\frac{\pi}{6}$

8. (8: 4,4) Compute:

a. $\cot(\csc^{-1} 3)$ let $\theta = \csc^{-1} 3$
 $\csc \theta = 3$

$\cot \theta = 2\sqrt{2}$

So, $\cot(\csc^{-1} 3) = 2\sqrt{2}$



b. $\sec(\tan^{-1} x)$. Note: Assume $x > 0$

let $\theta = \tan^{-1} x$

$\tan \theta = x$



$\sec \theta = \sqrt{x^2 + 1}$

So, $\sec(\tan^{-1} x) = \sqrt{x^2 + 1}$

9. (6) Prove: $\cot^2 x - \cos^2 x = \cos^2 x \cdot \cot^2 x$

$$\begin{aligned}
 \text{Pf } (\Rightarrow) \text{ LHS} &= \cot^2 x - \cos^2 x \\
 &= \frac{\cos^2 x}{\sin^2 x} - \cos^2 x \\
 &= \frac{\cos^2 x}{\sin^2 x} - \frac{\cos^2 x \sin^2 x}{\sin^2 x} \\
 &= \frac{\cos^2 x - \cos^2 x \sin^2 x}{\sin^2 x} \\
 &= \frac{\cos^2 x (1 - \sin^2 x)}{\sin^2 x} \\
 &= \frac{\cos^2 x}{\cancel{\sin^2 x}} \cdot \frac{\cos^2 x}{\sin^2 x} \\
 &= \cos^2 x \cdot \cot^2 x \\
 &= \text{RHS} \quad \square
 \end{aligned}$$

10. (7) Prove: $\frac{\sin x + \tan x}{\cot x + \csc x} = \sin x \cdot \tan x$

$$\begin{aligned}
 \text{Pf } (\Rightarrow) \text{ LHS} &= \frac{\sin x + \tan x}{\cot x + \csc x} \\
 &= \frac{\sin x + \frac{\sin x}{\cos x}}{\frac{\cos x}{\sin x} + \frac{1}{\sin x}} \cdot \frac{\sin x \cos x}{\sin x \cos x} \\
 &= \frac{\sin^2 x \cos x + \sin^2 x}{\cos^2 x + \cos x} \\
 &= \frac{\sin^2 x (\cos x + 1)}{\cos x (\cos x + 1)} \\
 &= \frac{\sin^2 x}{\cos x} \\
 &= \sin x \cdot \frac{\sin x}{\cos x} \\
 &= \sin x \cdot \tan x \\
 &= \text{RHS} \quad \square
 \end{aligned}$$

11. (8) Prove: $\frac{\cos x}{1 - \sin x} - \frac{\cos x}{1 + \sin x} = 2 \tan x$

$$\begin{aligned}
 \text{Pf } (\Rightarrow) \text{ LHS} &= \frac{\cos x}{1 - \sin x} - \frac{\cos x}{1 + \sin x} \\
 &= \frac{\cos x (1 + \sin x)}{(1 + \sin x)(1 - \sin x)} - \frac{\cos x (1 - \sin x)}{(1 + \sin x)(1 - \sin x)} \\
 &= \frac{\cos x + \cos x \sin x - \cos x + \cos x \sin x}{1 - \sin^2 x} \\
 &= \frac{2 \cos x \sin x}{\cos^2 x} \\
 &= \frac{2 \sin x}{\cos x} \\
 &= 2 \tan x \\
 &= \text{RHS} \quad \square
 \end{aligned}$$