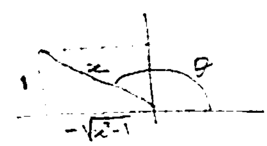
Summer 2012 Math 241 Test 1 Part 1: Non-Calculator Name: SOLUTION

Show all necessary work NEATLY, CLEARLY, and SYSTEMATICALLY for full point.

1. (9: 3-points each row) Write the values of sec θ, csc θ, and tan θ for the angles 0°, 30°, 45°, 60°, and 90°.

θ	45°	30°	0°	60°	90°
sec θ	$\sqrt{2}$	$\frac{2\sqrt{3}}{3}$	1	2	Undefined
csc θ	$\sqrt{2}$	2	Undefined	$\frac{2\sqrt{3}}{3}$	1
tan θ	1	$\frac{\sqrt{3}}{3}$	0	$\sqrt{3}$	Undefined

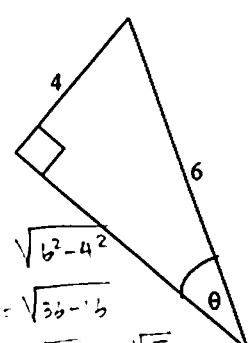
2. (5) Let $\csc\theta = x, \theta \in Q_2$. Find the values of the other 5 trigonometric functions of θ in terms of x.



$\sec\theta = -\frac{x}{\sqrt{x^2-1}} = -\frac{x\sqrt{x^2-1}}{x^2-1}$ $\sin\theta = \frac{1}{x}$

$\cos\theta = -\frac{\sqrt{x^2-1}}{x}$ $\tan\theta = -\frac{1}{\sqrt{x^2-1}} = -\frac{\sqrt{x^2-1}}{x^2-1}$ $\cot\theta = -\sqrt{x^2-1}$

3. (6: 1 each) Consider the following right triangle:



$\sqrt{6^2-4^2}$
$= \sqrt{36-16}$
$= \sqrt{20} = 2\sqrt{5}$

Find: (Note: You don't need to rationalize the denominators)

a. $\sin\theta = \frac{4}{6} = \frac{2}{3}$

b. $\cos\theta = \frac{2\sqrt{5}}{6} = \frac{\sqrt{5}}{3}$

c. $\tan\theta = \frac{4}{2\sqrt{5}} = \frac{2}{\sqrt{5}}$

d. $\sec\theta = \frac{6}{2\sqrt{5}} = \frac{3}{\sqrt{5}}$

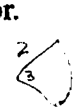
e. $\csc\theta = \frac{6}{4} = \frac{3}{2}$

f. $\cot\theta = \frac{2\sqrt{5}}{4} = \frac{\sqrt{5}}{2}$

4. (6:3 each) A sector formed by a central angle of 3 rad in a circle of radius 2 cm.

a. Find the perimeter of the sector.

P = "arc" + 2 "radius"



$= 2\cdot 3 + 2\cdot 2$

$= 10$ cm

b. Find the area of the sector.

$A = \frac{3}{2}\cdot 2^2$

$= 6$ cm^2

5. (8: 2 each) Compute and write the EXACT VALUES. Make sure you show logical steps leading to the answers.

a. $\sin(-135°)$

$= -\sin 135°$

$= -\sin 45°$

$= -\frac{\sqrt{2}}{2}$

b. $\tan(-120°)$

$= -\tan 120°$

$= -(-\tan 60°)$

$= \tan 60°$

$= \sqrt{3}$

c. $\cot 150°$

$= \frac{1}{\tan 150°}$

$= \frac{1}{-\tan 30°} = \frac{1}{-\frac{\sqrt{3}}{3}} = -\frac{3}{\sqrt{3}} = -3$

d. $\sec 225°$

$= \frac{1}{\cos 225°}$

$= \frac{1}{-\cos 45°}$

$= \frac{1}{-\frac{\sqrt{2}}{2}} = -\frac{2}{\sqrt{2}} = -\sqrt{2}$

6. (8: 2 each) Compute and write the EXACT VALUES. Make sure you show logical steps leading to the answers.

a. $\cos\left(-\frac{7\pi}{6}\right) = \cos \frac{7\pi}{6}$

$= -\cos \frac{\pi}{6}$

$= -\frac{\sqrt{3}}{2}$

b. $\tan\left(\frac{5\pi}{3}\right)$

$= -\tan \frac{\pi}{3}$

$= -\sqrt{3}$

c. $\csc\left(\frac{2\pi}{3}\right)$

$= \frac{1}{\sin\left(\frac{2\pi}{3}\right)}$

$= \frac{1}{\sin \frac{\pi}{3}} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

d. $\sec\left(\frac{3\pi}{2}\right)$

$= \frac{1}{\cos\left(\frac{3\pi}{2}\right)}$

$= \frac{1}{\cos\left(\frac{\pi}{2}\right)} = \frac{1}{0} =$ undefined

Summer 2012 Math 241 Test 1 Part 2: Calculator Name: SOLUTION

Show all necessary work NEATLY, CLEARLY, and SYSTEMATICALLY for full point.

7. (4:2,2) Use calculator to compute:

csc 110°

$= \frac{1}{\sin 110^\circ} = \frac{1}{0.93969}$

$= 1.06418$

cot 30

$= \frac{1}{\tan 30}$

$= \frac{1}{-6.40533} = -0.15612$

8. (4:2,2) Suppose $\theta \in Q_1$, use calculator to find the θ (in degree) such that:

cot θ = 3.45

$\tan\theta = \frac{1}{3.45}$

$\theta = \tan^{-1}\left(\frac{1}{3.45}\right) = 16.16457^\circ$

sec θ = 5.87

$\cos\theta = \frac{1}{5.87}$

$\theta = \cos^{-1}\left(\frac{1}{5.87}\right) = 80.19138^\circ$

9. (10: 1,1,1,2,2,3) Let $y = -3 - 2\sin(4x - \pi)$

a. Center $= -3$

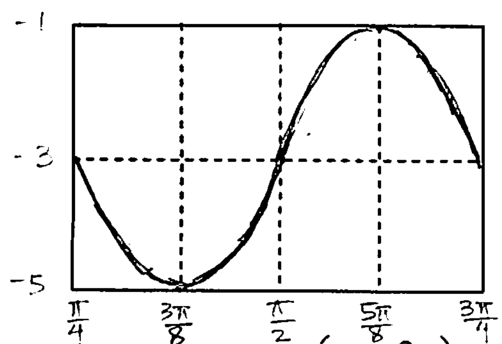
b. Amplitude $= 2$

c. period $= \frac{2\pi}{4} = \frac{\pi}{2}$

d. phase-shift $= \frac{\pi}{4}$

e. range $[-5, -1]$

f. Sketch the graph of one period in the provided window. Make sure you provide the details.



10. (10: 1,1,1,2,2,3) Let $y = -2 + 3\cos\left(4x + \frac{2\pi}{3}\right)$

a. Center $= -2$

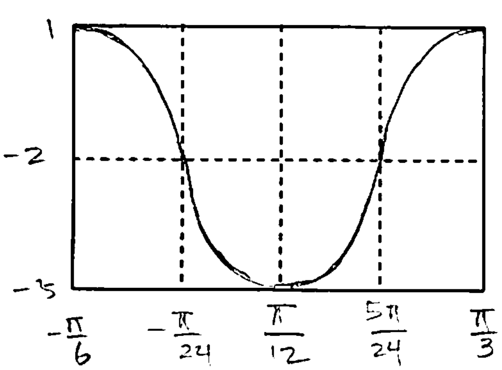
b. Amplitude $= 3$

c. period $\frac{2\pi}{4} = \frac{\pi}{2}$

d. phase-shift $= -\frac{\pi}{6}$

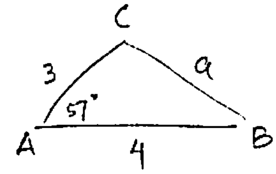
e. range $[-5, 1]$

f. Sketch the graph of one period in the provided window. Make sure you provide the details.



11. (8:4,4) Consider ΔABC, $A = 57°$, $b = 3$, $c = 4$.

a. Find a



$a^2 = b^2 + c^2 - 2bc \cos A$

$a^2 = 3^2 + 4^2 - 2 \cdot 3 \cdot 4 \cos 57°$

$a^2 = 11.92866$

$a = \sqrt{11.92866}$

$a = 3.45379$

b. Find B

$b^2 = a^2 + c^2 - 2ac \cos B$

$3^2 = 11.92866 + 4^2 - 2 \cdot 3.45379 \cdot 4 \cos B$

$9 = 27.92866 - 27.63032 \cos B$

$\cos B = 0.68507$

$B = 46.75900°$

Can also use Law of Sine

$\frac{\sin A}{a} = \frac{\sin B}{b}$

$\frac{\sin 57°}{3.45379} = \frac{\sin B}{3}$

$\sin B = \frac{3 \sin 57°}{3.45379}$

$\sin B = 46.75900°$

12. (8:4,4) Consider ΔABC, $a = 4$, $b = 6$, and $c = 8$.

a. Find A

$a^2 = b^2 + c^2 - 2bc \cos A$

$4^2 = 6^2 + 8^2 - 2 \cdot 6 \cdot 8 \cos A$

$16 = 36 + 64 - 96 \cos A$

$\frac{-84}{-96} = \cos A$

$A = 28.95502°$

b. Find area of ΔABC

Heron's formula:

$s = \frac{1}{2} P = \frac{1}{2}(4+6+8) = 9$

$A_\Delta = \sqrt{9(9-4)(9-6)(9-8)}$

$= \sqrt{9 \cdot 5 \cdot 3 \cdot 1}$

$= \sqrt{135} = 3\sqrt{15} = 11.61895$

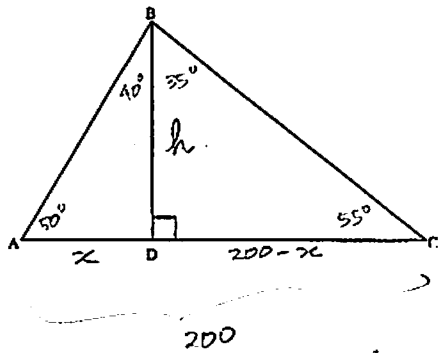
Can also use

$A_\Delta = \frac{1}{2} bc \sin A$

$= \frac{1}{2} \cdot 6 \cdot 8 \sin 28.95502°$

$= 11.61895$

13. (7) In ΔABC, $AC = 200$, $A = 50°$, $C = 55°$.
Find BD.
Note: You need to organize your work .



$$\tan 50° = \frac{h}{x} \rightarrow x = \frac{h}{\tan 50°}$$

$$\tan 55° = \frac{h}{200-x} \rightarrow 200 - x = \frac{h}{\tan 55°}$$

$$\rightarrow x + 200 - x = \frac{h}{\tan 50°} + \frac{h}{\tan 55°}$$

$$200 = h\left(\frac{1}{\tan 50°} + \frac{1}{\tan 55°}\right)$$

$$200 = h\left(\frac{\tan 55° + \tan 50°}{\tan 50° \tan 55°}\right)$$

$$h = 200\left(\frac{\tan 50° \tan 55°}{\tan 50° + \tan 55°}\right)$$

$$h = 129.92858$$

14. (13:6,3,4) Consider ΔABC, $B = 25°$, $b = 8$, $c = 16$.

a. Find C

$$\frac{\sin C}{c} = \frac{\sin B}{b}$$

$$\frac{\sin C}{16} = \frac{\sin 25°}{8}$$

$$\sin C = \frac{16 \sin 25°}{8}$$

$$\sin C = 0.84524$$

$$C_1 = 57.69729°$$

$$C_2 = 122.30271°$$

b. Find A

$$A_1 = 180° - (B + C_1)$$
$$= 180° - (25° + 57.69729°)$$
$$= 180° - 82.69729°$$
$$= 97.30271°$$

$$A_2 = 180° - (B + C_2)$$
$$= 180° - 147.30271° = 32.69729°$$

c. Find a

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$a = \frac{b \sin A}{\sin B}$$

(i) $$a_1 = \frac{8 \sin 97.30271°}{\sin 25°}$$

$$a_1 = 18.77606$$

(ii) $$a_2 = \frac{8 \sin 32.69729°}{\sin 25°}$$

$$a_2 = 10.22579$$