Winter 2014 Math 227 FINAL Name: SOLUTION

Show your work clearly, neatly, and understandably. Make sure you round the decimal for probability to 5-decimal place and round the percentage to 3-decimal.

1. (18:6,5,7) A company that produces cellphone batteries claims its battery lasts at least 35 hours on average. A consumer advocacy group questions that claim since a random sample of 7 batteries from that company has the lifetime of: 32, 35, 30, 31, 38, 34, and 31.

a. Find $\sum x$ and $\sum x^2$, then the sample mean and sample standard deviation.

$\sum x = 231$, $\sum x^2 = 7671$

$\bar{x} = 33$

$s^2 = \dfrac{7 \cdot 7671 - 231^2}{7 \cdot 6} = 8$

$s = 2.82843$

b. Assume that the lifetime of those batteries produced are normally distributed; construct at 99%-CI for the standard deviation of overall batteries lifetime.

① $df = 6$

① $\chi^2_R = 18.548$

① $\chi^2_L = 0.676$

① 99%-CI for σ^2:

$\dfrac{6 \cdot 8}{18.548} < \sigma^2 < \dfrac{6 \cdot 8}{0.676}$

$2.58788 < \sigma^2 < 71.00592$

② 99%-CI for σ:

$1.60869 < \sigma < 8.42650$

c. Assume that the lifetime of those batteries produced are normally distributed; test the company's claim at 0.05-SL.

1) Claim:

The battery's lifetime average is at least 35 hours.

2) Hypothesis:

$H_0: \mu \geq 35$

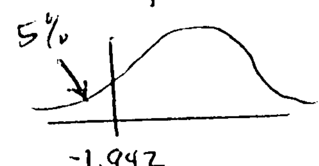
$H_1: \mu < 35$

3) Test-statistic

$t^* = \dfrac{33 - 35}{\frac{2.82843}{\sqrt{7}}} = -1.871$

4) Critical value $df = 6$

$t_c = -1.943$



5) Informal Conclusion

Fail to reject H_0

6) Conclusion:

At 5%-SL, the sample data is insufficient to reject the claim that the battery's lifetime average is at least 35 hours

2. (19:3,3,3,4,6) The length of calls to Amazon.com customer service is normally distributed with a mean of 15 min and a standard deviation of 4 min. Let X be the length of a call

a. What portion of calls is between 14.5 min and 17.2 min? $X \sim N(15, 4^2)$

$P(14.5 \le X \le 17.2)$

$= P\left(\frac{14.5-15}{4} \le Z \le \frac{17.2-15}{4}\right)$

$= P(-0.13 \le Z \le 0.55)$

$= 0.7088 - 0.4483 = \underline{0.2605}$

b. The percentage of calls longer than k minutes is 3%. Find *k*.

$P(X > k) = 3\%$

$P(X \le k) = 97\%$

$P(Z \le \frac{k-15}{4}) = 97\%$

$\frac{k-15}{4} = 1.88$

$k = 1.88 \cdot 4 + 15$

$k = \underline{22.52 \text{ min}}$

c. A sample of 16 calls is selected at random. Find the probability that less than 8 of them have length between 14.5 min and 17.2 min.

Let Y be the number of calls that is between 14.5 min and 17.2 min

$Y \sim B(16, 0.2605)$

$P(Y < 8) = P(Y \le 7)$

binomcdf(16, 0.2605, 7)

$= \underline{0.96564}$

d. Find the probability that the average length of 16 randomly selected calls is between 14.5 min and 17.2 min.

$\overline{X}_{16} \sim N(15, \frac{4^2}{16}) = N(15, 1)$

$P(14.5 \le \overline{X}_{16} \le 17.2)$

$= P\left(\frac{14.5-15}{1} \le Z \le \frac{17.2-15}{1}\right)$

$= P(-0.5 \le Z \le 2.2)$

$= 0.9861 - 0.3085$

$= \underline{0.6776}$

e. Find the probability that, of 100 calls, less than 80 with length between 14.5 min and 17.2 min. Find the actual probability and the Normal Approximation probability.

Let Y_2 be the number of calls that is between 14.5 min and 17.2 min

$Y_2 \sim B(100, 0.2605)$

$P(Y_2 < 80) = P(Y_2 \le 79)$

$= \underline{1}$

By NAtBD, $Y_2 \sim B(100, 0.2605) \doteq N(26.05, 19.264)$

$P(Y_2 < 80) \overset{cc}{=} P(Y_2 \le 79.5)$

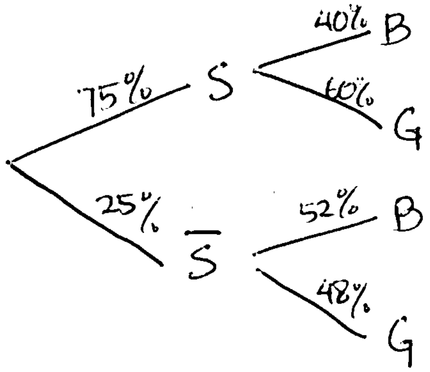
$= P\left(Z \le \frac{79.5 - 26.05}{4.3891}\right)$

$= P(Z \le 12.178)$

$= \underline{1}$

3. (14:3,3,4,4) In a certain school, 75% of the students play in soccer league. Of those who play in the league, 60% are girls. Of those who don't play in the league, 52% are boys.

a. Construct a probability tree diagram corresponds to the information above.



b. Find the probability that a randomly selected student is a boy.

$P(B) = P(S \cap B) + P(\bar{S} \cap B)$

$= 75\% \cdot 40\% + 25\% \cdot 52\%$

$= 0.75 \cdot 0.4 + 0.25 \cdot 0.52$

$= 0.3 + 0.13$

$= \underline{0.43}$

c. Find the probability that a randomly selected girl is in the league.

$P(S|G) = \frac{P(S \cap G)}{P(G)} = \frac{P(S) \cdot P(G|S)}{100\% - P(B)}$

$= \frac{75\% \cdot 60\%}{100\% - 43\%} = \frac{0.45}{0.57} = \frac{45}{57}$

d. Find the probability that, of 4 randomly selected girls, any of them are in the league.

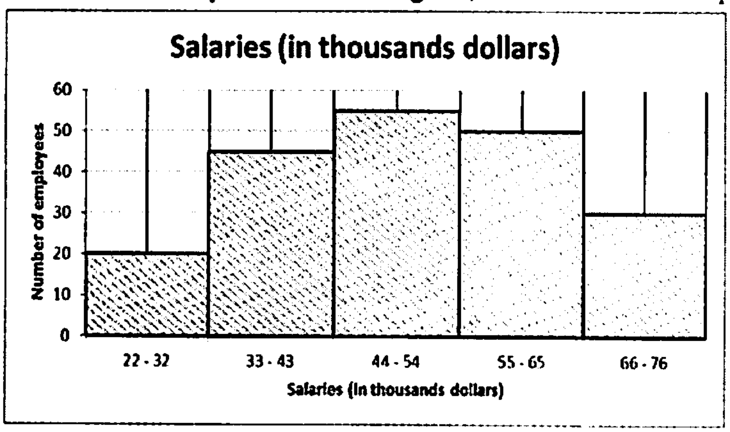
Let X be the number of girls in the league, $X \sim B(4, \frac{45}{57})$

$P(X \geq 1) = 1 - P(X=0)$

$= 1 - 0.00196$

$= 0.99804$

4. (10:3,3,4) The following is the histogram of the salaries of all the Acme Corporation employees. Construct a frequency distribution corresponds to the histogram, then extend it to compute the mean and standard deviation.



You can use normalization method.

x	f	x_i	fx_i	x_i^2	fx_i^2
22 - 32	20	27	540	729	14580
33 - 43	45	38	1710	1444	64980
44 - 54	55	49	2695	2401	132055
55 - 65	50	60	3000	3600	180000
66 - 76	30	71	2130	5041	151230
	200		10075		542845

$\mu = \frac{10075}{200} = \boxed{50.375}$

(ie, mean salary is \$50,375)

$\sigma^2 = \frac{200 \cdot 542845 - 10075^2}{200^2}$

$\sigma^2 = 176.58437$

$\sigma = \boxed{13.28851}$

(ie, standard deviation of salaries is \$13,289)

5. (11) A random sample of 25 male financial analysts' salaries resulted in a mean of $77,500 with a standard deviation of $6,000. Another random sample of 18 female financial analysts' salaries resulted in a mean of $72,000 with standard deviation of $9,000. With the assumption that both samples coming from normally distributed population, test the claim that average salary for female analysts is less than the average salary for male analysts. Use $\alpha = 0.05$.

Male	Female
$n_1 = 25$	$n_2 = 18$
$\bar{x}_1 = 77{,}500$	$\bar{x}_2 = 72{,}000$
$s_1 = 6000$	$s_2 = 9{,}000$

① Claim:
Average salary for female analysts is less than the average salary for male analysts.

② Hypothesis:

$H_0: \mu_2 \geq \mu_1 \quad (\mu_2 - \mu_1 \geq 0)$

$H_1: \mu_2 < \mu_1 \quad (\mu_2 - \mu_1 < 0)$

③ Test-statistic

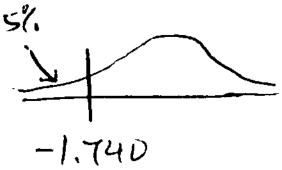
$$t^* = \frac{(\bar{x}_2 - \bar{x}_1) - (\mu_2 - \mu_1)}{\sqrt{\frac{s_2^2}{n_2} + \frac{s_1^2}{n_1}}}$$

$$t^* = \frac{72{,}000 - 77{,}500}{\sqrt{\frac{9000^2}{18} + \frac{6000^2}{25}}} = -2.257$$

④ Critical Value

use $df = 17$

$t_c = -1.740$



⑤ Informal Conclusion
Reject Ho

⑥ Conclusion
At 5% SL, the sample data supports the claim that the average salary for female analysts is less than the average salary for male analysts.

6. (11) A random sample of 175 residents in City A consists of 47 college graduates. Another random sample of 225 residents in City B consists of 81 college graduates. Do these cities have the same percentage of college graduates?

Test your claim at $\alpha = 0.03$.

City A	City B
$n_1 = 175$	$n_2 = 225$
$x_1 = 47$	$x_2 = 81$
$\hat{p}_1 = 0.26857$	$\hat{p}_2 = 0.36$

$$\bar{p} = \frac{47 + 81}{175 + 225} = \frac{128}{400} = 0.32$$

① Claim:
City A and City B have the same percentage of college graduates.

② Hypothesis

$H_0: p_1 = p_2 \quad (p_1 - p_2 = 0)$

$H_1: p_1 \neq p_2 \quad (p_1 - p_2 \neq 0)$

③ Test statistic

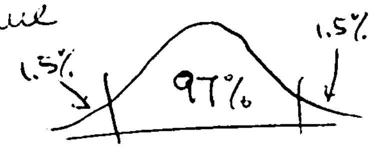
$$z^* = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\bar{p}\bar{q}}{n_1} + \frac{\bar{p}\bar{q}}{n_2}}}$$

$$z^* = \frac{0.26857 - 0.36}{\sqrt{\frac{0.32 \cdot 0.68}{175} + \frac{0.32 \cdot 0.68}{225}}}$$

$$z^* = \frac{-0.09143}{0.04702} = -1.9446$$

④ Critical Value

$z_c = \pm 2.17$



⑤ Informal conclusion
Fail to reject Ho

⑥ Conclusion
At 3%-SL, the sample data is sufficient to support the claim that City A and City B have the same percentage of college graduates.

7. (12:6,6) A clinic provides a program to help their clients lose weight and asks a consumer agency to investigate the effectiveness of the program. The agency takes a sample of 11 people, weighing each person in the sample before the program begins and 3 months later to produce the following data:

No.	Before	After	d	d^2
1	210	197	13	169
2	205	195	10	100
3	193	191	2	4
4	182	174	8	64
5	259	236	23	529
6	239	226	13	169
7	222	201	21	441
8	211	196	15	225
9	187	181	6	36
10	243	229	14	196
11	246	231	15	225
			140	2158

a. Fill the table above and compute the mean and standard deviation of the differences.

$\bar{d} = \frac{140}{11} = 12.72727$

$s_d^2 = \frac{11 \cdot 2158 - 140^2}{11 \cdot 10} = 37.61818$

$s_d = 6.13337$

b. Construct a 95%-CI for the mean of all weight loss.

① $df = 10$ $\quad t_{\frac{\alpha}{2}} = 2.228$

② $E = 2.228 \cdot \frac{6.13337}{\sqrt{11}}$

$E = 4.12020$

③ 95%-CI for μ_d:

$12.72727 - 4.12020 \leq \mu_d \leq 12.72727 + 4.12020$

$8.60707 \leq \mu_d \leq 16.84747$

8. (8:5,3) A box contains of 4 red, 3 green, and 2 yellow marbles, all of which are identical except in color. Three marbles are randomly selected without repetition from the box. Construct the probability distribution and extend to find the expected number of red marbles selected.

Let X be the number of red marbles selected.

4R	5$\bar{R}$

$X = \{0,1,2,3\}$

X	P	PX
0	$\frac{{}_4C_0 \cdot {}_5C_3}{{}_9C_3} = \frac{1\cdot 10}{84} = \frac{5}{42}$	0
1	$\frac{{}_4C_1 \cdot {}_5C_2}{{}_9C_3} = \frac{4\cdot 10}{84} = \frac{20}{42}$	$\frac{20}{42}$
2	$\frac{{}_4C_2 \cdot {}_5C_1}{{}_9C_3} = \frac{6\cdot 5}{84} = \frac{15}{42}$	$\frac{30}{42}$
3	$\frac{{}_4C_3 \cdot {}_5C_0}{{}_9C_3} = \frac{4\cdot 1}{84} = \frac{2}{42}$	$\frac{6}{42}$
	$\frac{42}{42} = 1$	$\frac{56}{42} = \frac{4}{3}$

So, $E(X) = \frac{4}{3}$