Winter 2014 Math 227 Test 1 Name: SOLUTION

Show all necessary work clearly, neatly, systematically, and for full-points. Since part of this is a Test on Descriptive Statistics, the aesthetics aspect of your presentation is considered part of my grading. There are 105 points available

The following are the means SAT Math scores from 50 states and DC in 2004. The data are sorted

476	507	515	543	566	602
493	509	519	546	573	
495	510	521	547	576	
499	512	523	553	585	
499	514	524	553	585	
499	514	(528)	555	593	
501	514	531	556	596	
502	514	539	557	597	
502	514	539	557	597	
506	515	542	561	601	

From the data, find:

$\frac{51+1}{2} = 26$ → 26th data

1. (2) Median = 528
2. (2) Mode = 514
3. (3) Q_1 = 510 $\frac{25+1}{2} = 13$ → 13th data
4. (3) Q_3 = 557
5. (2) Range = 602 − 476 = 126

6. (6) Create a Frequency Distribution with 5 classes.

$w = \frac{126}{5} = 25.2 \approx 26$

X	f
476 − 501	7
502 − 527	18
528 − 553	10
554 − 579	8
580 − 605	8
	51

7. (8) COPY the Frequency Distribution in #6 below and EXTEND with lower class limits, upper class limits, lower class boundaries, upper class boundaries, and relative frequency.

X	f	lcl	ucl	lcb	ucb	rf
476 − 501	7	476	501	475.5	501.5	$\frac{7}{51}$
502 − 527	18	502	527	501.5	527.5	$\frac{18}{51}$
528 − 553	10	528	553	527.5	553.5	$\frac{10}{51}$
554 − 579	8	554	579	553.5	579.5	$\frac{8}{51}$
580 − 605	8	580	605	579.5	605.5	$\frac{8}{51}$
	51					$\frac{51}{51} = 1$

8. (10) COPY the Frequency Distribution in #6 below and EXTEND to find the estimated mean and standard deviation. Round to 2 decimal-places. (*) use normalized method

x	f	x_i^*	fx_i^*	x_i^{*2}	fx_i^{*2}
476-501	7	-2	-14	4	28
502-527	18	-1	-18	1	18
528-553	10	0	0	0	0
554-579	8	1	8	1	8
580-605	8	2	16	4	32
	51		-8		86

$$\sigma^{2*} = \frac{51.86 - (-8)^2}{51^2}$$

$$\sigma^{2*} = \frac{4322}{2601} = 1.66167$$

$$\sigma^* = 1.28906$$

$$\sigma = 26 \times 1.28906$$

$$\sigma = 33.52$$

$$\mu^* = \frac{-8}{51}$$

$$\mu = 540.5 + 26\left(-\frac{8}{51}\right)$$

$$= 536.42$$

9. (4) From #8, what is the interval within 2 standard deviations of the mean? And list the outliers.

$\mu + 2\sigma = 536.42 + 2 \times 33.52 = 603.46$

$\mu - 2\sigma = 536.42 - 2 \times 33.52 = 469.38$

Usual interval: (469.38, 603.46)

Outlier: NONE

10. (6:3,3) Suppose the height 40 year-old females have mean 5.5 ft with standard deviations 0.4. According to Chebychev, what is the minimum portion of the population has height between 4.9 ft and 6.1 ft. You need to first find how many standard deviations are these from the mean.

$\mu = 5.5$, $\sigma = 0.4$

$$z = \frac{4.9 - 5.5}{0.4} = -1.5$$

$$z = \frac{6.1 - 5.5}{0.4} = 1.5$$

So, 1.5 σ away from the mean

By Chebychev,

at least $1 - \frac{1}{1.5^2} = 0.55556$

of the population.

11. (8:5,3) Suppose P(A) = 0.5, P(B) = 0.7, and P(A∪B) = 0.9. Find:

a. P(A∩B)

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.9 = 0.5 + 0.7 - P(A \cap B)$

$0.9 = 1.2 - P(A \cap B)$

$P(A \cap B) = 1.2 - 0.9$

$P(A \cap B) = 0.3$

b. P(A|B)

$$= \frac{P(A \cap B)}{P(B)}$$

$$= \frac{0.3}{0.7}$$

$$= \frac{3}{7}$$

12. (8:5,3) Suppose P(A) = 0.5, P(B) = 0.6, and P(A|B) = 0.75.

a. P(A∩B)

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$0.75 = \frac{P(A \cap B)}{0.6}$$

$$0.75 \times 0.6 = P(A \cap B)$$

$$0.45 = P(A \cap B)$$

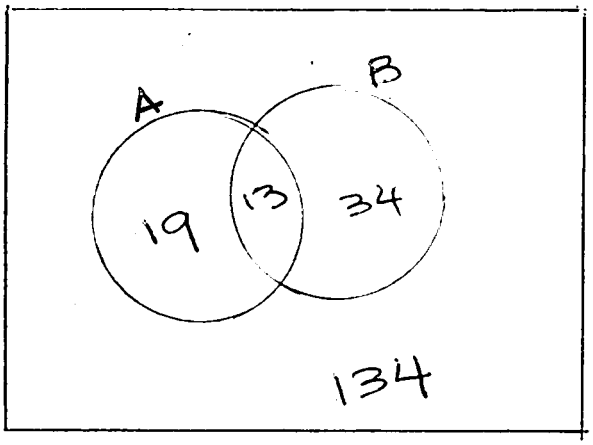
b. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.5 + 0.6 - 0.45$

$= 0.65$

13. (21:5,3,3,5,5) In a company of 200 employees, there are 32 employees making at least \$100,000/year. Also, 47 of those 200 employees have a graduate degree. Let A be the event "making at least \$100,000/year" and B be the event "having a graduate degree". There are 13 employees making at least \$100,000/year and having a graduate degree

a. Create a Venn Diagram.



b. Write the notation and find the probability that a randomly selected employee makes less than \$100,000/year and has a graduate degree.

$$P(\bar{A} \cap B) = \text{[shaded region]}$$

$$\frac{34}{200} = 0.17$$

c. Write the notation and find the probability that a randomly selected employee makes less than \$100,000/year or has a graduate degree.

$$P(\bar{A} \cup B) = \frac{181}{200} = 0.905$$

d. Write the notation and find the probability that an employee with a graduate degree makes less than \$100,000/year.

$$P(\bar{A}|B) = \frac{34}{47} = 0.72340$$

e. TWO employees are randomly selected from those 200 employees to attend a conference. Find the probability that BOTH have a graduate degree.

$$P(B_1 B_2) = \frac{47}{200} \cdot \frac{46}{199}$$

$$= 0.05432$$

14. (8) List 4 levels of measurements and provide 3 examples for each level.

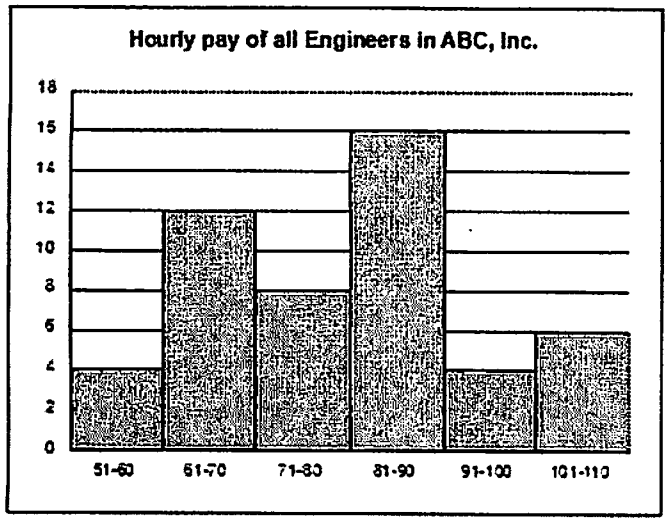
1) Nominal, Ex: Car brand, Gender, Country, Color

2) Ordinal, Ex: Satisfaction level, military rank, income level, lettergrade

3) Interval, Ex: Temperature, Time during the day, latitude

4) Ratio, Ex: length, time, weight

15. (14:6,8) Consider the following histogram:



a. Create the Frequency Distribution from the histogram.

Hourly Pay	Frequency
51-60	4
61-70	12
71-80	8
81-90	16
91-100	4
101-110	6
	50

b. Extend to find the estimated mean and standard deviation

X	f	x_i	fx_i	x_i^2	fx_i^2
51-60	4	55.5	222	3080.25	12321
61-70	12	65.5	786	4290.25	51483
71-80	8	75.5	604	5700.25	45602
81-90	16	85.5	1368	7310.25	116964
91-100	4	95.5	382	9120.25	36481
101-110	6	105.5	633	11130.25	66781.5
	50		3995		329632.5

$\mu = \frac{3995}{50} = 79.9$

$\sigma^2 = \frac{50 \times 329632.5 - 3995^2}{50^2}$

$\sigma^2 = \frac{521600}{2500} = 208.64$

$\sigma = 14.444388$