Winter 2014 Math 227 Test 2 Name: SOLUTION

Show your work clearly, neatly, and understandably. Make sure you round the decimal for probability to 4-decimal place and round the percentage to 2-decimal. There are 103 points available.

1. (Total 24) According to Gallup Poll conducted between January 14-19, 2014, Obama's approval rating among all voters is 39% and his disapproval rating is 53%, while the rest have no opinion.

   a. (3) Of 15 voters, find the expected number of voters approving Obama.

   Let X_1 be the number of voters approving Obama.

   $X_1 \sim B(15, 0.39)$

   $E(X_1) = 15 \times 0.39 = 5.85$

   b. (4) Of 25 voters, find the standard deviation of the number of voters disapprove Obama.

   Let X_2 be the number of voters disapproving Obama.

   $X_2 \sim B(25, 0.53)$

   $\sigma(X_2) = \sqrt{25 \times 0.53 \times 0.47} = \sqrt{6.2275}$

   $= 2.4955$

   c. (5) Find the probability that, of 8 people, more than 5 of them approve Obama.

   Let X_3 be the number of voters approving Obama.

   $X_3 \sim B(8, 0.39)$

   $P(X_3 > 5) = 1 - P(X_3 \le 5)$ $\boxed{\text{binomcdf}}$

   $= 1 - 0.9561$

   $= 0.0439$

   d. (5) Find the probability that any of those 8 people has/have no opinion.

   Let X_4 be the number of voters having no opinion

   $X_4 \sim B(8, 0.08)$

   $P(X_4 \geq 1) = 1 - P(X_4 = 0)$

   $= 1 - 0.5132$

   $= 0.4868$

   e. (7) Of 50 voters, find the "usual" interval for the number of voters having no opinion.

   Let X_5 be the number of voters having no opinion.

   $X_5 \sim B(50, 0.08)$

   $\mu = 50 \times 0.08 = 4$

   $\sigma = \sqrt{50 \times 0.08 \times 0.92} = \sqrt{3.68}$

   $= 1.9183$

   $\mu + 2\sigma = 7.8367$

   $\mu - 2\sigma = 0.1633$

   So, the usual interval is $(0.1633, 7.837)$

2. (Total 19) A box contains 12 identical cards (except in colors): 5 red, 4 blue, and 3 green.

   a. (4) Three cards are selected without replacement. Find the probability of selecting any green cards.

   $\boxed{3G \mid 9\bar{G}}$

   $P(G \geq 1) = 1 - P(G = 0)$

   $= 1 - \frac{{}_3C_0 \cdot {}_9C_3}{{}_{12}C_3}$

   $= 1 - \frac{1 \cdot 84}{220}$

   $= 1 - \frac{84}{220}$

   $= \frac{136}{220} = \frac{34}{55}$

   b. (5) A card is drawn randomly 5 times with replacement. Find the probability of getting a red card three times.

   Let X be the number of times a red card selected.

   $X \sim B(5, \frac{5}{12})$

   $P(X = 3) = {}_5C_3 \cdot (\frac{5}{12})^3 (\frac{7}{12})^2$

   $= 0.24615$

   $= 0.2462$

c. (10) Create a probability distribution of the number of blue cards selected for a procedure of selecting three cards without replacement. Let X be the number of blue cards selected.

$X = \{0, 1, 2, 3\}$

$| 4B | 8\bar{B} |$

x	P
0	$\frac{{}_4C_0 \cdot {}_8C_3}{{}_{12}C_3} = \frac{1 \cdot 56}{220} = \frac{56}{220}$
1	$\frac{{}_4C_1 \cdot {}_8C_2}{{}_{12}C_3} = \frac{4 \cdot 28}{220} = \frac{112}{220}$
2	$\frac{{}_4C_2 \cdot {}_8C_1}{{}_{12}C_3} = \frac{6 \cdot 8}{220} = \frac{48}{220}$
3	$\frac{{}_4C_3 \cdot {}_8C_0}{{}_{12}C_3} = \frac{4 \cdot 1}{220} = \frac{4}{220}$
	$\frac{220}{220} = 1$

3. (Total 15:6,3,6) For each million tickets sold, the original New York Lottery awarded one \$50,000 prize, nine \$5,000 prizes, ninety \$500 prizes, and nine hundred \$50 prizes. Construct the probability distribution of the payout then extend to find its expected value and standard deviation. Let X be the payout

x	P	xP	x^2	x^2P
50000	$\frac{1}{1000000}$	$\frac{50000}{1000000}$	2500000000	2500
5000	$\frac{9}{1000000}$	$\frac{45000}{1000000}$	25000000	225
500	$\frac{90}{1000000}$	$\frac{45000}{1000000}$	250000	22.5
50	$\frac{900}{1000000}$	$\frac{45000}{1000000}$	2500	2.25
0	$\frac{999000}{1000000}$	0	0	0
	1	$\frac{185000}{1000000}$		2749.75

$E(X) = \frac{185000}{1000000} = \underline{\underline{0.185}}$

$Var(X) = 2749.75 - 0.185^2$

$= 2749.7158$

$\sigma(X) = \underline{\underline{52.4377}}$

2h
2t
2a

4. (4) Write your full name in one word: thomasharjuno Ex: thomasharjuno.

Find the number of different "passwords" can be created by permuting all the letters in your full name.

$$\frac{13!}{2!2!2!} = 778{,}377{,}600$$

5. (Total 12) Super Lotto Plus Lottery draws 5 "regular" numbers out of 47 in which the order does not matter AND 1 "mega" number out of 27. Find the probability to correctly select: (Just use the notation, do not compute into decimal).

$$\underbrace{|5C|42W|}_{\text{Regular}} \quad \underbrace{|1C|26W|}_{\text{Mega}}$$

a. (4) None of all six numbers.

$$\frac{{}_5C_0 \cdot {}_{42}C_5}{{}_{47}C_5} \cdot \frac{{}_1C_0 \cdot {}_{26}C_1}{{}_{27}C_1}$$

b. (4) Three "regular" numbers but not the "mega" number.

$$\frac{{}_5C_3 \cdot {}_{42}C_2}{{}_{47}C_5} \cdot \frac{{}_1C_0 \cdot {}_{26}C_1}{{}_{27}C_1}$$

c. (4) One "regular" and the "mega" numbers.

$$\frac{{}_5C_1 \cdot {}_{42}C_4}{{}_{47}C_5} \cdot \frac{{}_1C_1 \; {}_{26}C_0}{{}_{27}C_1}$$

6. (Total 12) A proposed legislation has 0.76 probabilities to pass in the House, 0.62 in the Senate, and 0.89 on at least one of them. $P(H) = 0.76$, $P(S) = 0.62$ $P(H \cup S) = 0.89$

a. (4) Find the probability to pass both.

$$P(H\cup S) = P(H) + P(S) - P(H\cap S)$$
$$0.89 = 0.76 + 0.62 - P(H\cap S)$$
$$0.89 = 1.38 - P(H\cap S)$$
$$P(H\cap S) = 1.38 - 0.89$$

$P(H\cap S) = \underline{0.49}$

b. (3) Find the probability to pass none.

$$P(\text{none}) = P(\text{neither}) = 1 - P(H\cup S)$$
$$= 1 - 0.89$$
$$= \underline{0.11}$$

c. (5) Given a legislation passed the House, find the probability it will also pass the Senate.

$$P(S|H) = \frac{P(H\cap S)}{P(H)}$$
$$= \frac{0.49}{0.76}$$
$$= \underline{0.6447}$$

7. (Total 17) A random sample of 50 purchases at a store produced the following contingency table for payment method and the size of purchase.

	Cash	*Credit Card*	*Debit Card*	
Under \$30	5	2	2	9
\$30 - \$100	2	10	9	21
Over \$100	1	12	7	20
	8	24	18	50

a. (3) Find the probability that a purchase is at most \$100.

$P(\text{at most } \$100) = \frac{30}{50} = 0.6$

b. (3) Find the probability that a purchase is paid not by a credit card.

$P(\text{not by cc}) = 1 - P(\text{by cc}) = 1 - \frac{24}{50} = \frac{26}{50} = 0.52$

c. (5) Given that a purchase is paid by a debit card, find the probability that it was at least \$30.

$P(\text{at least } \$30 \mid DC) = \frac{16}{18} = \frac{8}{9}$

d. (6) Given that a credit card purchase was at least \$30, find the probability that is was over \$100.

$P(\text{over } \$100 \mid (CC \cap \text{at least } \$30))$

$= \frac{12}{22}$

$= \frac{6}{11}$