Winter 2014 Math 227 Test 3 Name: SOLUTION

Show your work clearly, neatly, and understandably. Make sure you round the decimal for probability to 5-decimal place and round the percentage to 3-decimal.

1. (Total 25) Women's heights are normally distributed with a mean of 63.5 inches and a standard deviation of 2.5 inches.

   a. (3) What percentages of women are between 62 inches and 65 inches?

   Let X be the height of a woman.

   $X \sim N(63.5, 2.5^2)$

   $P(62 \le X \le 65)$

   $= P\left(\frac{62-63.5}{2.5} \le Z \le \frac{65-63.5}{2.5}\right)$

   $= P(-0.6 \le Z \le 0.6)$

   $= 0.7257 - 0.2743$

   $= 0.4514 = 45.14\%$

   b. (4) The percentage of women taller than k inches is 7%. Find *k*.

   $P(X > k) = 7\%$

   $P(X \le k) = 93\%$

   $P\left(Z \le \frac{k-63.5}{2.5}\right) = 93\%$

   $\frac{k-63.5}{2.5} = 1.48$

   $k = 1.48 \times 2.5 + 63.5$

   $k = 67.2$ in

   c. (4) A sample of 9 women is selected at random. Find the probability that 7 or 8 of them have height between 62–65 inches.

   Let X_2 be the number of woman with height between 62 to 65 in.

   $X_2 \sim B(9, 0.4514)$

   $P(X_2 = 7 \text{ or } 8) = P(X_2 = 7) + P(X_2 = 8)$

   $= 0.04138 + 0.00851$

   $= 0.04989$

d. (7) Find the probability that the average height of 9 randomly selected women is between 62–65 in.

$\bar{X}_9 \sim N(63.5, \frac{2.5^2}{9})$

$P(62 \le \bar{X}_9 \le 65)$

$= P\left(\frac{62-63.5}{\frac{2.5}{\sqrt{9}}} \le Z \le \frac{65-63.5}{\frac{2.5}{\sqrt{9}}}\right)$

$= P(-1.8 \le Z \le 1.8)$

$= 0.9641 - 0.0359$

$= 0.9282$

e. (7) Find the probability that, of 150 women, more than 100 with height between 62–65 in.

First method

Normal Approximation

Let X_3 be the number of women with height between 62 to 65 in

$X_3 \sim B(150, 0.4514) \doteqdot N(67.71, 37.14571)$

$P(X_3 > 100) \overset{cc}{\doteqdot} P(X_3 > 100.5)$

$= P\left(Z > \frac{100.5 - 67.71}{\sqrt{37.14571}}\right)$

$\boxed{\begin{array}{l} np = 67.71 > 5 \\ nq = 82.29 > 5 \end{array}}$

$= P(Z > 5.38)$

$\doteqdot 0.0001$ (by textbook's table)

$\doteqdot 0$ (by calculator)

Second method

$P(X_3 > 100)$

$= 1 - P(X_3 \le 100)$

$= 1 -$ binomcdf$(150, 0.4514, 100)$

$= 1 - 0.9999999648$

$= 0.0000000352$

$\doteqdot 0$

2. (10) A machine is programmed to put 737 grams of sugar into a container. Due to uncontrolled variation in the process, there is variation in content from container to container. To estimate the mean amount of sugar per container, a sample of 50 boxes is selected and its mean is 739.5 grams. If σ = 7.5 grams. Construct a 95%-confidence interval for μ.

$n = 50$

$\bar{x} = 739.5$

$\sigma = 7.5$

CL = 95%

(a) $Z_{\frac{\alpha}{2}} = 1.96$

(b) $E = 1.96 \frac{7.5}{\sqrt{50}}$

$E = 2.07889$

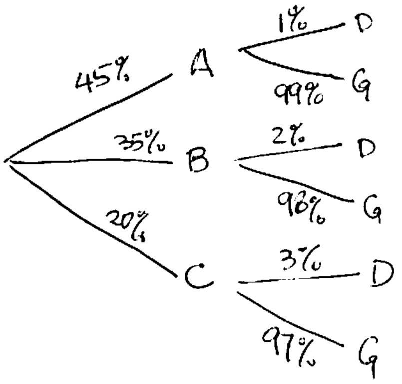
(c) 95%-CI for μ:

$739.5 - 2.07889 < \mu < 739.5 + 2.07889$

$737.42111 < \mu < 741.57889$

3. (Total 20) The chips of a computer manufacturer are supplied by 3 companies: 45% from A, 35% from B, and the rest by C. Of those supplied by A, 1% are defective; by B, 2% are defective; by C, 3% are defective.

a. (4) Construct the Tree Diagram of the situation.



b. (5) A chip is randomly selected. Find the probability that the chip is defective.

$P(D) = P(A \cap D) + P(B \cap D) + P(C \cap D)$

$= 45\% \cdot 1\% + 35\% \cdot 2\% + 20\% \cdot 3\%$

$= 0.45 \times 0.01 + 0.35 \times 0.02 + 0.20 \times 0.03$

$= 0.0045 + 0.0070 + 0.0060$

$= \underline{0.0175} = \underline{1.75\%}$

c. (4) Given a chip is defective. Find the probability that the chip is from C.

$P(C|D) = \dfrac{P(C \cap D)}{P(D)}$

$= \dfrac{0.0060}{0.0175}$

$= \dfrac{60}{175} = \underline{\dfrac{12}{37}} = \underline{0.34286}$

d. (7) Out of 150 defective chips, find the probability that more than 50 are from company C. Note: I want you to use Normal Approximation to Binomial Distribution here.

Let X be the number of defective chips from C.

$X \sim B(150, 0.34286) \cong N(51.42857, 33.79592)$

$np = 51.42857 > 5$

$nq = 98.57143 > 5$

$P(X > 50) \overset{cc}{=} P(X > 50.5)$

$= P\left(Z > \dfrac{50.5 - 51.42857}{\sqrt{33.79592}}\right)$

$= P(Z > -0.16)$

$= 1 - P(Z < -0.16)$

$= 1 - 0.4364$

$= \underline{0.5636}$

4. (13: 7,4,2) We would like to start an ISP (Internet Service Provider) and need to estimate the average internet usage of households in one week for our business plan and model. Assume that a previous survey of household usage has shown $\sigma = 36.95$ minutes. Assume the distribution of internet usage is normally distributed. 95% CL

a. (7) If the desired margin of error 3 minutes. Find the minimum sample size.

$\sigma = 36.95$

$E = 3$

$CL = 95\% \rightarrow z = 1.96$

$$n = \frac{z_{\alpha/2}^2 \cdot \sigma^2}{E^2} = \frac{1.96^2 \times 36.95^2}{3^2} = 582.77 \longrightarrow \text{Use } n = 583$$

b. (4) If the desired margin of error is 1 minutes. Find the minimum sample size.

$E = 1$

$$\rightarrow n = \frac{1.96^2 \times 36.95^2}{1^2} = 5244.95 \longrightarrow \text{use } n = 5245$$

c. (2) State the relationship between your answer in part (a) and part (b): The minimum sample size in part (b) is (approximately) ___9___ times the minimum sample size in part (a).

5. (Total 15) The number of times a student uses the Library Quite Room is described by the following table:

Number of monthly usage	*Probability*	px	x^2	px^2
0	0.05	0	0	0
1	0.25	0.25	1	0.25
2	0.50	1.00	4	2.00
3	0.20	0.60	9	1.80
		1.85		4.05

a. (2) Find the expected number of monthly usage of Library Quiet Room by a student.

$E(X) = 1.85$

b. (5) Find the standard deviation for the number of monthly usage of Library Quiet Room by a student.

$\sigma^2 = 4.05 - 1.85^2$

$\sigma^2 = 0.6275$

$\sigma = \sqrt{0.6275} = 0.79215$

c. (8) In a sample of 100 students, find the approximate probability that the average number of monthly usage of Library Quite Room is at most 1.8. (per student)

Let X be the number of monthly usage for a student, $X \sim \mu = 1.85, \sigma^2 = 0.6275$

Since $n > 100$, $\bar{X}_{100} \sim N(1.85, \frac{0.6275}{100})$ ← by CLT

$P(\bar{X}_{100} \leq 1.8)$

$= P\left(Z \leq \frac{1.8 - 1.85}{\sqrt{\frac{0.6275}{100}}}\right)$

$= P(Z \leq -0.20) = \underline{\underline{0.4207}}$

6. (Total 17) A state licensing exam that is given annually has been designed such that the scores are normally distributed with mean 68 and standard deviation 15.

a. (4) What is the percentage of the scores between 65 and 89?

Let X be the exam score $X \sim N(68, 15^2)$

$P(65 < X < 89)$

$= P\left(\frac{65-68}{15} < Z < \frac{89-68}{15}\right)$

$= P(-0.2 < Z < 1.4)$

$= 0.9192 - 0.4207$

$= \underline{\underline{0.4985}}$

b. (6) If 45% of the test-takers passed the test and licensed, what is the lowest passing score?

$P(X > k) = 0.45$

$\Rightarrow P(X \leq k) = 0.55$

$P\left(Z \leq \frac{k-68}{15}\right) = 0.55$

$\rightarrow \frac{k-68}{15} = 0.13$

$k = 0.13 \times 15 + 68$

$k = \underline{\underline{69.95}}$

c. (7) If there are 650 people take the test, find the probability that more than 300 of them pass the test.

Let X_1 be the number of people pass the test.

$X_1 \sim B(650, 0.45)$

$P(X_1 > 300) = 1 - P(X_1 \leq 300)$

$= 1 - \text{binomcdf}(650, 0.45, 300)$

$= 1 - 0.73614$

$= \underline{\underline{0.26386}}$

NOTE: You can also use NA to BD to approximate this