Winter 2014 Math 227 Test 4 Name: SOLUTION

Show your work clearly, neatly, systematically and understandably. Round the decimal for probability to 5-decimal place and round the percentage to 3-decimal. Use proper notation.

1. (10:3,3,4) **Find the critical values.**
   a. **Assume that the normal distribution applies. Find the critical value.**
      Right-tailed test; $\alpha = 0.04$. 0.96 0.04 $z = 1.75$
   b. **Assume that t-distribution applies. Find the critical value.**
      Left-tailed test; $\alpha = 0.01$, and $n = 21$. $df = 20$ 0.01 $t = -2.528$
   c. **Assume that the chi-square distribution applies. Find the critical values.**
      Two-tailed test; $\alpha = 0.02$, $n = 21$. 0.01 0.01 $\chi^2_L = 8.260$ $\chi^2_R = 37.566$

2. (11:4,7) **A consumer advocacy group will conduct a survey to find the proportion p of consumers who were happy with their iPhone 5 purchase.**
   a. **Find the sample size they need to estimate p with 2% margin of error and 90% confidence?**

No $\hat{p}$ provided, so use $\hat{p} = \hat{q} = 0.5$

$E = 2\% = 0.02$

$CL = 90\% \rightarrow z_{\frac{\alpha}{2}} = 1.645$

$n = \dfrac{1.645^2 \times 0.5 \times 0.5}{0.02^2}$

$n = 1691.27$

Use $n = 1692$

   b. **The advocacy group took a random sample of 1000 consumers who recently purchased iPhone 5 and found that 400 were happy with their purchase. Construct a 95% confidence interval for p.**

$n = 1000 \rightarrow \hat{p} = 0.4$

$x = 400 \quad \hat{q} = 0.6$

$CL = 95\% \rightarrow z_{\frac{\alpha}{2}} = 1.96$

1. $E = 1.96\sqrt{\dfrac{0.4 \times 0.6}{1000}}$

$E = 1.96 \cdot 0.0154919$

$E = 0.03036$

2. 95%-CI for p:

$0.4 - 0.03036 \le p \le 0.4 + 0.03036$

$0.36963 \le p \le 0.43036$

3. (19:6,6,7) A random sample of size 15 is taken from the population known to be normally distributed. From the sample, it is found that $\sum x = 1203, \sum x^2 = 97565$.

a. Find the sample mean and sample standard deviation.

$n = 15$

$\bar{x} = \frac{1203}{15} = 80.2$

$s^2 = \frac{15 \cdot 97565 - 1203^2}{15 \cdot 14}$

$s^2 = 77.45714$

$\rightarrow s = 8.80097$

b. Construct 99% confidence interval for μ.

$CL = 99\%$

$df = 14$

① $t_{\frac{\alpha}{2}} = 2.977$

② $E = 2.977 \cdot \frac{8.80097}{\sqrt{15}}$

$E = 6.76494$

③ 99%-CI for μ:

$80.2 - 6.76494 \leq \mu \leq 80.2 + 6.76494$

$73.43506 \leq \mu \leq 86.96494$

c. Construct 95% confidence interval for σ.

$CL = 95\%$

$df = 14$

① $\chi^2_L = 5.629$

$\chi^2_R = 26.119$

② 95%-CI for σ^2:

$\frac{14 \cdot 77.45714}{26.119} \leq \sigma^2 \leq \frac{14 \cdot 77.45714}{5.629}$

$41.5176676 \leq \sigma^2 \leq 192.645223$

③ 95%-CI for σ:

$6.44342 \leq \sigma \leq 13.87967$

4. (10) A car insurance company is reviewing its current policy rates. When originally setting the rates, they believed that the average loss amount was $1,800. They are concerned that the true mean is actually higher than this, because then they could potentially lose a lot of money. They randomly select 40 accidents and find a sample mean of $1,950 loss. Assuming that the standard deviation of overall loss is $500. Test, at 5% SL, to see if the insurance company should be concerned.

$n = 40$, $\bar{x} = \$1950$, $\sigma = 500$

$SL = 0.05$

① Claim: The true loss mean is higher than $1800

② Hypothesis:

$H_0: \mu \leq 1800$

$H_1: \mu > 1800$

③ Test-statistic

$$z^* = \frac{1950 - 1800}{\frac{500}{\sqrt{40}}} = 1.90$$

④ Critical Value

$z_c = 1.645$

⑤ Informal Conclusion

Reject H_0

⑥ Conclusion

At 5%-SL, the sample data is sufficient to support the claim that the true loss mean is higher than $1800

5. (10) A sample of 40 sales receipts from a grocery store has mean of $137 and standard deviation $30.2. Use these values to test, at 0.05-SL, whether or not the true mean of sales at the grocery store are different from $150.

$n = 40$, $\bar{x} = \$137$, $s = 30.2$

$SL: 0.05$ $df = 39$

① Claim. The true mean of sales at the grocery store is different from $150.

② Hypothesis

$H_0: \mu = 150$

$H_1: \mu \neq 150$

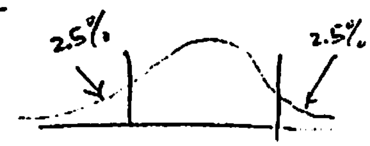
③ Test-statistic

$$t^* = \frac{137 - 150}{\frac{30.2}{\sqrt{40}}} = -2.722$$

④ Critical value

$t_c = \pm 2.024$

for $df = 38$



⑤ Informal conclusion

Reject H_0

[Note: we will still reject H_0 if we use $df = 40$ for t_c.

⑥ Conclusion

At 5% SL, the sample data appears to support the claim that the true mean of sales at the grocery store is different from $150.

6. (10) In 1996, 25% of students who had perfect attendance one month would also have perfect attendance the following month. In 2000, the school wants to see if the proportion has changed. The proportion of a random sample of 6543 students is 23.4%. At 5% SL, should the school conclude that there has been a change?

$n = 6543$ $\quad$ SL = 5%

$\hat{p} = 23.4\%$

1) Claim: The proportion of students who had perfect attendance one month that would also have perfect attendance in the following month has been different from 25%

2) Hypothesis:

$H_0: p = 0.25$

$H_1: p \neq 0.25$

3) Test-statistic

$$z^* = \frac{0.234 - 0.25}{\sqrt{\frac{0.25 \times 0.75}{6543}}}$$

$z^* = -2.99$

4) Critical value

$z_c = \pm 1.96$

5) Informal Conclusion

Reject Ho

6) Formal Conclusion

At 5% SL, the sample data is sufficient to support the claim that the proportion of students who had perfect attendance one month ~~that~~ would also have perfect attendance in the following month has been different from 25%

7. (10) A company operates a bottle-filling machine. They decide to replace the machine if the variance of the amount dispensed per bottle is greater than 250 ml with 0.01 SL. On a particular day, a random sample of 15 bottles yields a mean of 357.2 ml and a standard deviation of 16.9 ml. Should the machine be replaced?

SL = 0.01 $\quad$ $s = 16.9$

$\bar{x} = 357.2$ $\quad$ $n = 15$

1) Claim: The bottle-filling machine should be replaced

[or, the variance of the amount dispensed per bottle is greater than 250 ml]

2) Hypothesis

$H_0: \sigma^2 \leq 250$

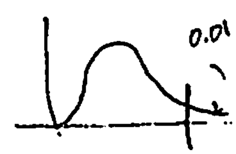
$H_1: \sigma^2 > 250$

3) Test-statistic

$$\chi^{2*} = \frac{14 \cdot 16.9^2}{250} = 15.994$$

4) Critical value

$\chi^2_c = 29.141$



5) Informal Conclusion

Fail to reject Ho

6) Formal conclusion

At 0.01-SL, there is insufficient information to support the claim that the bottle-filling machine should be replaced

8. (20:10,10) **We want to compare ground versus air-based temperature sensors to determine the earth's temperature, which is important for agricultural modeling, etc. Ground-based sensors are expensive, and air-based (from satellites or air-planes) of infrared wavelengths may be biased. We collected temperature data from ground and air-based sensors at ten locations, and the sampling resulted in the following:**

d = "Ground" − "Air"

Location	Ground (in °C)	Air (in °C)	d	d²
1	46.9	47.3	−0.4	0.16
2	45.4	48.1	−2.7	7.29
3	36.3	37.9	−1.6	2.56
4	31	32.7	−1.7	2.89
5	24.7	26.2	−1.5	2.25
6	22.3	23.3	−1.0	1.00
7	49.8	50.2	−0.4	0.16
8	40.5	42.6	−2.1	4.41
9	37.7	39.4	−1.7	2.89
10	35.5	37.9	−2.4	5.76
			−15.5	29.37

a. **Fill up the table above. Then compute mean of differences and standard deviation of differences.**

$\Sigma d = -15.5$

$\Sigma d^2 = 29.37$

$n = 10$

$\bar{d} = \frac{-15.5}{10} = -1.55$

$s_d^2 = \frac{10 \cdot 29.37 - (-15.5)^2}{10 \cdot 9}$

$s_d^2 = \frac{293.7 - 240.25}{90}$

$s_d^2 = \frac{53.45}{90} = 0.59389$

$s_d = 0.77064$

b. **Test, at 0.05-SL, if the air-based sensors measurement is significantly more than ground-based sensors measurement.**

① Claim: The air-based sensors measurement is significantly more than ground-based measurement

② Hypothesis

$H_0: \mu_d \geq 0$

$H_1: \mu_d < 0$

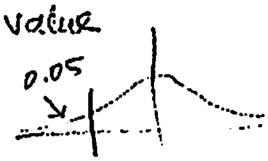
③ Test-statistic

$t^* = \frac{-1.55 - 0}{\frac{0.77064}{\sqrt{10}}}$

$t^* = -6.360$

④ Critical value

$t_c = -1.833$



⑤ Informal conclusion

Reject H_0

⑥ Conclusion

At 0.05 SL, the sample data is sufficient to support the claim that the air-based sensors measurement is significantly more than ground-based sensors measurement.

9. (7) **In some states the law requires drivers to turn on their headlights when driving in the rain. A highway patrol officer believes that only between one-quarter and one-third (inclusive) of all drivers follow this rule. He wants to construct a 98% confidence interval for the proportion of all drivers who turns on their headlights when driving in the rain. How many cars driving in the rain does he need to examine so that this 98% confidence interval for the proportion has a width of no more than seven percentage points?**

$\frac{1}{4} \leq \hat{p} \leq \frac{1}{3} \longrightarrow$ use $\hat{p} = \frac{1}{3}$, so $\hat{q} = \frac{2}{3}$

$CL = 98\% \longrightarrow z_{\frac{\alpha}{2}} = 2.33$

width $= 7\% = 0.07 \longrightarrow E = \frac{0.07}{2} = 0.035$

$$n = \frac{2.33^2 \times \frac{1}{3} \times \frac{2}{3}}{0.035^2}$$

$n = 984.83$

Use $\underline{\underline{n = 985}}$