Note: ① I show my work here by writing on the exam.
Make sure you adjust your way to show work on scratch paper so that you do not write on the exam.
② I tried to show my work using textbook way.
I will include "my way" but it is only for students with better comprehension

So: $l_1: 3x-5y=5$ and $l_2 \perp l_1$ → $l_2: -5x-3x=C_2$ → $(5,1) \to -5(5)-3(1)=C_2$, $-25-3=C_2$, $-28=C_2$, $\to -5x-3x=-28$

NOTE: In general form IF $l_1: Ax+By=C_1$ and $l_2 \perp l_1$ THEN $l_2: Bx-Ay=C_2$

DEPARTMENTAL FINAL EXAM—MATH 125 FALL08

VERSION 1 **TEST#_____**

DO NOT WRITE ON EXAM

---

1. **Write an equation of the line through the point (5, 1) that is perpendicular to the line $3x-5y=5$.**

A) $3x-5y=10$
B) $-5x+3y=-22$
C) $3x+5y=20$
D) $-3x+5y=5$
E) $-5x-3y=-28$

$l_1: 3x-5y=5$
$-5y=-3x+5$
$y=\frac{-3}{-5}x+\frac{5}{-5}$
$y=\frac{3}{5}x-1$
$\to m_1=\frac{3}{5}$
$l_1 \perp l_2 \Rightarrow m_2=-\frac{5}{3}$

So: $l_2: y=-\frac{5}{3}x+b$
$(5,1) \to 1=-\frac{5}{3}(5)+b$
$1=-\frac{25}{3}+b$
$1+\frac{25}{3}=b$
$\frac{3}{3}+\frac{25}{3}=b$
$\frac{28}{3}=b$

$l_2: y=-\frac{5}{3}x+\frac{28}{3}$
$3y=-5x+28$
$5x+3y=28$
$-5x-3y=-28$ (E)

---

2. **Multiply and simplify.**

$\frac{4}{5+8x}\cdot(45+72x)$

A) 36, $x \neq -\frac{5}{8}$
B) 32, $x \neq -\frac{8}{5}$
C) 20, $x \neq 0$
D) 45, $x \neq \frac{5}{8}$
E) 40, $x \neq \frac{8}{5}$

$\frac{4}{5+8x}\cdot\frac{45+72x}{1}$
$=\frac{4}{5+8x}\cdot\frac{9(5+8x)}{1}$ factorize and cross-cancel
$=\frac{36}{1}$
$=36$ (A)

$5+8x \neq 0$
$8x \neq -5$
$x \neq -\frac{5}{8}$

---

3. **Divide and simplify.**

$\frac{x^2+100}{9(x+5)} \div \frac{x+10}{9(x^2-25)}$

A) $\frac{(x+5)(x^2+100)}{81(x+10)}$, $x \neq 5$
B) $\frac{x+10}{x+5}$, $x \neq 9$, $x \neq 10$
C) $(x-5)(x-10)$, $x \neq -9$, $x \neq -5$
D) $\frac{(x-5)(x^2+100)}{x+10}$, $x \neq -5$
E) $\frac{9(x-5)}{x-10}$, $x \neq -9$, $x \neq 10$

$\frac{x^2+100}{9(x+5)} \div \frac{x+10}{9(x^2-25)}$ "flip" and factorize
$=\frac{x^2+100}{9(x+5)}\cdot\frac{9(x+5)(x-5)}{x+10}$
$=\frac{(x^2+100)(x-5)}{x+10}$ (D)

$x+5 \neq 0$
$x \neq -5$

Textbook: $A(t) = A_0 e^{-kt}$

Halftime constant $k = -\frac{\ln 2}{t_0}$

$t_0 = 5730 \rightarrow k = -\frac{\ln 2}{5730}$

So $A(t) = A_0 \cdot e^{-\frac{t \ln 2}{5730}}$

$A(1000) = A_0 e^{-\frac{1000 \ln 2}{5730}}$

$3 = A_0 e^{-\frac{1000 \ln 2}{5730}}$

$3 \cdot e^{\frac{1000 \ln 2}{5730}} = A_0$

$3.39 \cong A_0$

4. Radioactive carbon, ^{14}C, has a half-life of 5730 years. After 1000 years, 3.00 grams remain. What was the initial quantity of ^{14}C? Round your answer to two decimal places.

A) 3.39 grams
B) 3.30 grams
C) 5.66 grams
D) 3.16 grams
E) 3.52 grams

Halftime is given, as 5730 years so use $A(t) = A_0 2^{-\frac{t}{5730}}$

when $t = 1000$, $A(1000) = 3$

$\rightarrow A(1000) = A_0 2^{-\frac{1000}{5730}}$

$3 = A_0 \cdot 2^{-\frac{1000}{5730}}$

$3 \cdot 2^{\frac{1000}{5730}} = A_0$

$3.386 \cong A_0$

$3.39 \cong A_0$

(A)

---

5. Use the Quadratic Formula to solve $12x^2 - 12x + 3 = 0$.

A) $x = -\frac{1}{2}$
B) $x = 12\sqrt{2}$
C) $x = \frac{1}{2}$
D) $x = \frac{1}{2} \pm 12\sqrt{2}$
E) no solutions

$a = 12, b = -12, c = 3$

Discriminant: $D = b^2 - 4ac = (-12)^2 - 4(12)(3)$

$= 144 - 144$

$= 0$

$\rightarrow x_{1,2} = \frac{-(-12) \pm \sqrt{0}}{2 \cdot 12}$

$x_{1,2} = \frac{12 \pm 0}{24}$

$x_{1,2} = \frac{12}{24} = \frac{1}{2}$ (C)

NOTE:
$12x^2 - 12x + 3 = 0$
$3(4x^2 - 4x + 1) = 0$
$3(2x-1)(2x-1) = 0$
$2x - 1 = 0 \rightarrow x = \frac{1}{2}$

IF you think about it, when $D = 0$, then

① The equation is not only factorable (since D is a perfect square), moreover,

② $x_{1,2} = -\frac{b}{2a}$ (since $D = 0$)

---

6. Find the domain of the rational function.

$h(t) = \frac{2t^2}{t^2 - 3t - 54}$

A) $(-\infty, -6) \cup (-6, 9) \cup (9, \infty)$
B) $(-\infty, 0) \cup (0, \infty)$
C) $(-\infty, 0) \cup (0, 9) \cup (9, \infty)$
D) $(-\infty, \infty)$
E) $(-\infty, -6) \cup (-6, 0) \cup (0, \infty)$

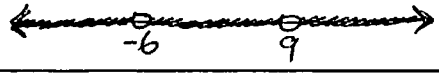
$h(t) = \frac{2t^2}{t^2 - 3t - 54}$

Dom $h(t) = \{t \mid t^2 - 3t - 54 \neq 0\}$

$\rightarrow t^2 - 3t - 54 \neq 0$

$(t - 9)(t + 6) \neq 0$

$t \neq 9 \quad t \neq -6$



(A)

---

7. Find $(f \circ g)(x)$ where $f(x) = 3x - 5$ and $g(x) = \sqrt{x + 9}$.

A) $(3x - 5)\sqrt{x + 9}$
B) $\sqrt{3x + 27} - 5$
C) $3\sqrt{x + 4}$
D) $\sqrt{3x + 4}$
E) $3\sqrt{x + 9} - 5$

$(f \circ g)(x)$

$= f(g(x))$

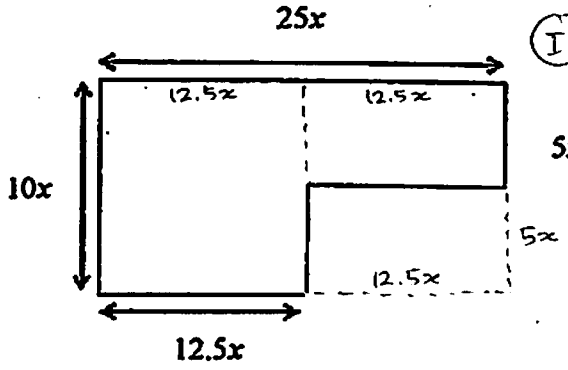
$= f(\sqrt{x+9})$

$= 3(\sqrt{x+9}) - 5$

$= 3\sqrt{x+9} - 5$

(E)

8. Find an expression for the area of the figure. Assume all angles are right angles.



(I) Area $= 12.5x \cdot 10x + 12.5x \cdot 5x$
$= 125x^2 + 62.5x^2$
$= 187.5x^2$ (B)

(II) Area $= 25x \cdot 10x - 12.5x \cdot 5x$
$= 250x^2 - 62.5x^2$
$= 187.5x^2$

A) $125x$
B) $187.5x^2$
C) $125x^2$
D) $70x$
E) $187.5x$

---

9. Use the properties of logarithms to condense $7\log_{11} x - 5\log_{11} z$.

A) $\log_{11}\frac{x^7}{z^5}$
B) $\log_{11} x^5 - \log_{11} z^5$
C) $\log_{11}(x-z)^2$
D) $\log_{11}(7x-5z)$
E) $\log_{11}\frac{7x}{5z}$

$7\log_{11} x - 5\log_{11} z$
$= \log_{11} x^7 - \log_{11} z^5$
$= \log_{11}\left(\frac{x^7}{z^5}\right)$ (A)

---

10. Find the intercepts of $y = 7x - x^2$.

A) x-intercepts: $(0,0)$ and $\left(\frac{1}{7},0\right)$
y-intercept: $(0,0)$
B) x-intercepts: $(-7,0)$ and $(7,0)$
y-intercept: $(0,0)$
C) x-intercept: $(0,0)$
y-intercepts: $(0,0)$ and $(7,0)$
D) x-intercepts: $(0,0)$ and $(-7,0)$
y-intercept: $(0,0)$
E) x-intercepts: $(0,0)$ and $(7,0)$
y-intercept: $(0,0)$

$y = 7x - x^2$

y-int $x = 0 \longrightarrow y = 7(0) - 0^2$
$y = 0$
$\longrightarrow$ y-int $(0,0)$

x-int $y = 0 \longrightarrow 0 = 7x - x^2$
$0 = x(7-x)$
$x = 0$, $7 - x = 0$
$x = 7$

$\rightarrow$ x-int: $(0,0)$ and $(7,0)$
(E)

11. Solve $|2x-1| \le 7$, if possible. Write the answer in set notation.

A) $\{x|-4 \le x \le 3\}$

B) no solution

C) $\{x|-\frac{7}{2} \le x \le \frac{7}{2}\}$

D) $\{x|-3 \le x \le 4\}$

E) $\{x|-3 < x < 4\}$

$|2x-1| \le 7$

$\rightarrow -7 \le 2x-1 \le 7$

$-6 \le 2x \le 8$

$-3 \le x \le 4$

(D)

---

12. Find the inverse function.

$f(x) = 9-3x$

A) $f^{-1}(x) = \frac{x-9}{3}$

B) $f^{-1}(x) = \frac{1}{3}x+9$

C) The function does not have an inverse.

D) $f^{-1}(x) = \frac{9-x}{3}$

E) $f^{-1}(x) = -\frac{1}{3}x-9$

$f(x) = 9-3x$

① $y = 9-3x$

② $x = 9-3y$

③ $x-9 = -3y$

$\frac{x-9}{-3} = y$

$\frac{9-x}{3} = y$

④ $f^{-1}(x) = \frac{9-x}{3}$

① Change $f(x)$ to y

② Swap x and y

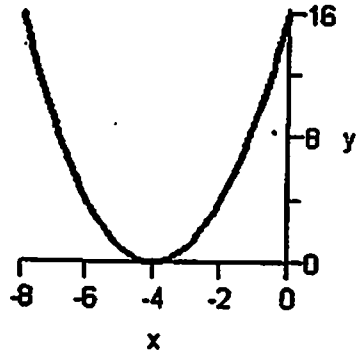
③ Solve for y

④ Change y to $f^{-1}(x)$

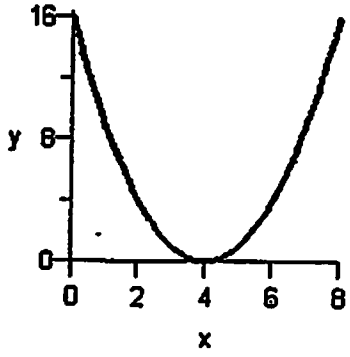
---

13. Sketch the graph of $h(x) = -(x-4)^2$.

A)



B)



① $a < 0$

→ open downward

② $h(x) = -(x-4)^2 + 0$

vertex $(4,0)$

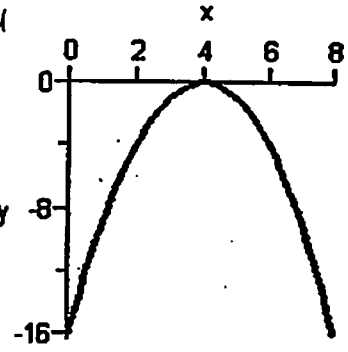
③ $h(0) = -(-4)^2$

$= -16$

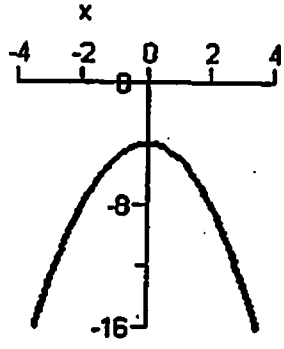
→ y-int $(0,-16)$

(C)

C)



D)



14. Use synthetic division to divide.

$$\frac{x^3+3x^2-11}{x+4}$$

A) $x^2+x+2-\frac{17}{x+4}$

B) $x^2-x-\frac{7}{x+4}$

C) $x^2-x+4-\frac{27}{x+4}$

D) $x^2+7x+28+\frac{123}{x+4}$

E) $x^2-x+6-\frac{12}{x+4}$

$$\frac{x^3+3x^2+0x-11}{x+4}$$

$$\begin{array}{r|cccc} -4 & 1 & 3 & 0 & -11 \\ & & -4 & 4 & -16 \\ \hline & 1 & -1 & 4 & -27 \end{array}$$

$\rightarrow x^2-x+4-\frac{27}{x+4}$

(C)

---

15. Perform the division.

$$(x^2+9x+20)\div(x+5)$$

A) $x+9+\frac{5}{x+5}$, $x\neq -5$

B) $x+5$, $x\neq 5$

C) $x+1+\frac{16}{x+5}$, $x\neq 5$

D) $x+4$, $x\neq -5$

E) $x+\frac{20}{x+5}$, $x\neq -5$

$$\frac{x^2+9x+20}{x+5}$$

$$= \frac{(x+5)(x+4)}{x+5}$$

$$= x+4, \quad x\neq -5$$

(D)

---

16. Simplify the expression.

$$\frac{(a-5)^{\frac{1}{7}}}{\sqrt[7]{a-5}}$$

A) $(a-5)^{\frac{2}{7}}$

B) $(a-5)^6$

C) $\frac{1}{(a-5)^{\frac{2}{7}}}$

D) -1

E) 1

NOTE $\sqrt[n]{x} = x^{\frac{1}{n}}$

So, $\sqrt[7]{a-5} = (a-5)^{\frac{1}{7}}$

$$\rightarrow \frac{(a-5)^{\frac{1}{7}}}{\sqrt[7]{a-5}} = \frac{(a-5)^{\frac{1}{7}}}{(a-5)^{\frac{1}{7}}} = 1, \quad a\neq 5$$

(E)

17. Graph the system of linear inequalities below.

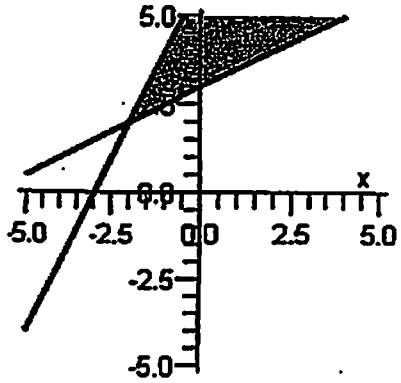
$$\begin{cases} y \geq \frac{1}{2}x+1 & \text{--- (1)} \\ y \leq 2x-2 & \text{--- (2)} \end{cases}$$

(1) $y \geq \frac{1}{2}x+1$
$m = \frac{1}{2}$
y-int $(0,1)$
shade above

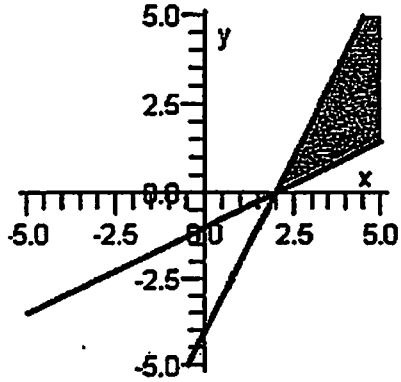
(2) $y \leq 2x-2$
$m = 2$
y-int $(0,-2)$
shade below

(E)

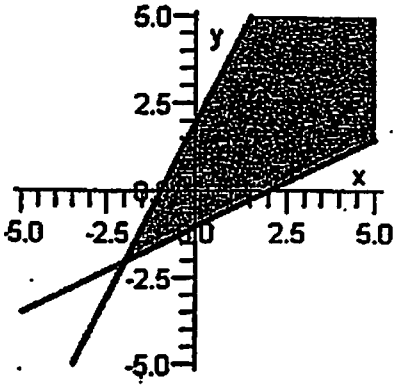
A)



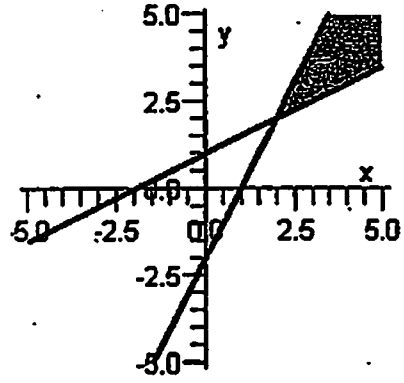
D)



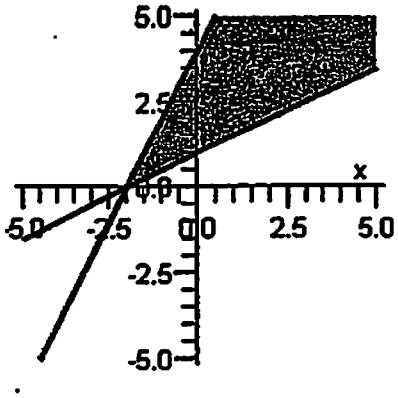
B)



E)



C)



18. Describe the domain of the function.

$g(x)=\sqrt{8-3x}$

A) $x \geq \frac{3}{8}$

B) $x \geq \frac{8}{3}$

C) $x \leq \frac{8}{3}$

D) all real numbers

E) $x \leq \frac{3}{8}$

$\text{Dom } g(x) = \{x \mid 8-3x \geq 0\}$

$8-3x \geq 0$

$8 \geq 3x$

$\frac{8}{3} \geq x$

(C)

---

19. Rewrite the expression using rational exponents.

$\frac{\sqrt[3]{x^3}}{\sqrt[3]{x^{10}}}$

$= \sqrt[3]{\frac{x^3}{x^{10}}} = \sqrt[3]{\frac{1}{x^7}} = \frac{1}{\sqrt[3]{x^7}} = \frac{1}{x^{\frac{7}{3}}}$ (A)

A) $\frac{1}{x^{\frac{7}{3}}}$

B) $\frac{1}{x^7}$

C) x^7

D) x^3

E) $\frac{1}{x^3}$

---

20. Use the rules of exponents to simplify the expression below.

$\left[\frac{(2x^3)(2x)^2}{(-2x)(2x)}\right]^3$

$= \left[\frac{2x^3 \cdot 4x^2}{-4x^2}\right]^3$

$= \left[\frac{8x^5}{-4x^2}\right]$

$= (-2x^3)^3$

$= -8x^9$ (E)

A) $-128x^{13}$

B) $-8x^{13}$

C) $-64x^{15}$

D) $-x^9$

E) $-8x^9$

21. Identify the horizontal asymptote of $g(x) = 4^{x+5}$.

A) $y = 1024$

B) $y = \frac{1}{4}$

C) x-axis

D) $y = -5$

E) $y = 5$

$g(x) = 4^{x+5}$

is from $g_0(x) = 4^x$ being shifted horizontally 5 units to the left

Since horizontal asymptote of $g_0(x)$ is $y = 0$

the horizontal asymptote of $g(x)$ is also $y = 0$ (C)

---

22. A chemist wishes to mix a 30% acid solution with a 55% acid solution to obtain 120 quarts of a 40% acid solution. How many quarts of each solution should be mixed together?

A) 72 quarts of 30% solution
48 quarts of 55% solution

B) 66 quarts of 30% solution
54 quarts of 55% solution

C) 60 quarts of 30% solution
60 quarts of 55% solution

D) 36 quarts of 30% solution
84 quarts of 55% solution

E) 54 quarts of 30% solution
66 quarts of 55% solution

Let x be the amount of 55% acid solution

(so, the amount of 30% acid solution is $120 - x$)

The amount of "pure" acid:

$55\% \cdot x + 30\%(120 - x) = 40\% \cdot 120$

$0.55x + 36 - 0.3x = 48$

$0.25x + 36 = 48$

$0.25x = 12$

$x = \frac{12}{0.25}$

$x = 48$ (A)

So, The amount of 55% acid solution is 48 quarts, and 30% is 72 quarts

---

23. Expand $(x+5)^3$ and simplify.

A) $x^3 + 5x^2 + 25x + 125$

B) $x^3 - 75x^3 + 15x - 125$

C) $x^3 + 125$

D) $x^3 + 75x^2 + 15x + 125$

E) $x^3 + 15x^2 + 75x + 125$

$(x+5)^3 = (x+5)(x+5)(x+5)$

$= (x^2 + 10x + 25)(x+5)$

$= x^3 + 5x^2 + 10x^2 + 50x + 25x + 125$

$= x^3 + 15x^2 + 75x + 125$

(E)

---

24. The height h (in feet) of an object dropped from a tower 64 feet high is modeled by $h = 64 - 16t^2$, where t is the time in seconds. How long does it take for the object to reach the ground? Round your answer to two decimal places.

A) 2.00 seconds
B) 4.57 seconds
C) 0.50 seconds
D) 6.93 seconds
E) 4.00 seconds

to reach ground $\rightarrow h = 0$

$0 = 64 - 16t^2$

$16t^2 = 64$

$t^2 = 4$

$t = \pm 2$

$t_1 = 2$ $\quad t_2 = -2$ N/A

(A)

25: Solve the system of linear equations below.

$$\begin{cases} 2x-3y+4z=14 \cdots (1) \\ 4x+y-3z=-5 \cdots (2) \\ 3x+2y-2z=-3 \cdots (3) \end{cases}$$

A) $(0,3,1)$
B) $(0,1,3)$
C) $(3,1,0)$
D) $(3,0,1)$
E) $(1,0,3)$

(1)&(2): $2x-3y+4z=14$ | $\times 1$; $4x+y-3z=-5$ | $\times 3$

$2x-3y+4z=14$
$12x+3y-9z=-15$ (+)
$14x-5z=-1$ (4)

(2)&(3): $4x+y-3z=-5$ | $\times(-2)$; $3x+2y-2z=-3$ | $\times 1$

$-8x-2y+6z=10$
$3x+2y-2z=-3$ (+)
$-5x+4z=7$ (5)

(4)&(5): $14x-5z=-1$ | $\times 4$; $-5x+4z=7$ | $\times 5$

$56x-20z=-4$
$-25x+20z=35$ (+)
$31x=31$

So, $x=1$

(4) $\rightarrow 14-5z=-1$
$-5z=-15$
$z=3$

(2) $\rightarrow 4+y-9=-5$
$y-5=-5$
$y=0$

So, $(x,y,z)=(1,0,3)$

E

---

26. Solve $x=\sqrt{44-7x}$, if possible.

A) no solution
B) $x=4$
C) $x=44$
D) $x=7$
E) $x=-4$

$x=\sqrt{44-7x}$
$x^2=44-7x$
$x^2+7x-44=0$
$(x+11)(x-4)=0$
$x+11=0 \quad x-4=0$
$x=-11 \quad x=4$

Check

$x=-11$: $-11 \stackrel{?}{=} \sqrt{44-7(-11)}$
$-11 \stackrel{?}{=} \sqrt{121}$
$-11=11$ X

$x=4$: $4 \stackrel{?}{=} \sqrt{44-7(4)}$
$4 \stackrel{?}{=} \sqrt{16}$
$4=4$ ✓

So, $x=4$ (B)

---

27. Combine and simplify.

$$\frac{x+2}{x-1}-\frac{2}{x+10}-\frac{22}{x^2+9x-10}$$

A) $\frac{x(x-10)}{(x-1)(x+10)}$, $x\neq 10$
B) $\frac{x}{x-1}$, $x\neq -10$
C) $\frac{x+10}{x+9}$, $x\neq 10$
D) $\frac{x-22}{(x-1)(x+10)}$, $x\neq -10$
E) $\frac{x-1}{x+10}$, $x\neq -10$

$$\frac{x+2}{x-1}-\frac{2}{x+10}-\frac{22}{(x+10)(x-1)}$$

$$=\frac{(x+2)(x+10)}{(x-1)(x+10)}-\frac{2(x-1)}{(x-1)(x+10)}-\frac{22}{(x+10)(x-1)}$$

$$=\frac{(x^2+12x+20)-2(x-1)-22}{(x-1)(x+10)}$$

$$=\frac{x^2+12x+20-2x+2-22}{(x-1)(x+10)}$$

$$=\frac{x^2+10x}{(x-1)(x+10)}$$

$$=\frac{x(x+10)}{(x-1)(x+10)}$$

$$=\frac{x}{x-1}, \quad x\neq -10$$

(B)

28. Solve the equation. Check for extraneous solutions.

$$\frac{5}{x+10}+\frac{2}{x^2+2x-80}=\frac{-4}{x-8}$$

A) $x=\frac{2}{9}$

B) $x=-\frac{20}{9}$

C) $x=-2$

D) $x=2$

E) $x=-\frac{2}{9}$

$$\frac{5}{x+10}+\frac{2}{(x+10)(x-8)}=\frac{-4}{x-8}$$

$\times (x+10)(x-8)$

$$5(x-8)+2=-4(x+10)$$
$$5x-40+2=-4x-40$$
$$5x-38=-4x-40$$
$$5x+4x=-40+38$$
$$9x=-2$$
$$x=-\frac{2}{9}$$

(E)

---

29. Write the quotient in standard form.

$$\frac{3i}{5+4i}$$

A) $-\frac{12}{41}+\frac{15}{41}i$

B) $\frac{15}{41}-\frac{12}{41}i$

C) $-\frac{15}{41}-\frac{12}{41}i$

D) $\frac{12}{41}+\frac{15}{41}i$

E) none of the above

$$\frac{3i}{5+4i}=\frac{3i}{5+4i}\cdot\frac{5-4i}{5-4i}$$
$$=\frac{3i(5-4i)}{25+16}$$
$$=\frac{15i+12}{41}$$
$$=\frac{12}{41}+\frac{15}{41}i$$

(D)

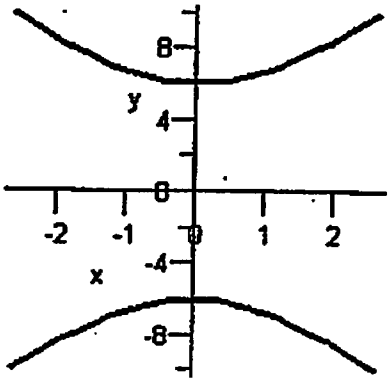
30. Graph the hyperbola.

$\frac{x^2}{4} - \frac{y^2}{36} = 1$

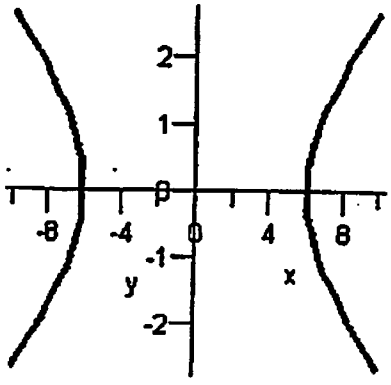
$\frac{x^2}{2^2} - \frac{y^2}{6^2} = 1$

→ hiperbola
① opens left-right
② "center" origin
③ vertices
(2,0), (-2,0)
(C)

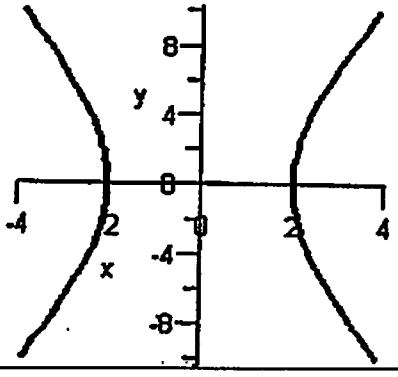
A)



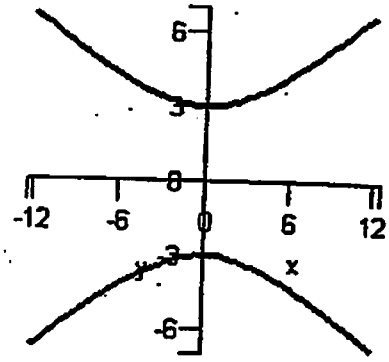
B)



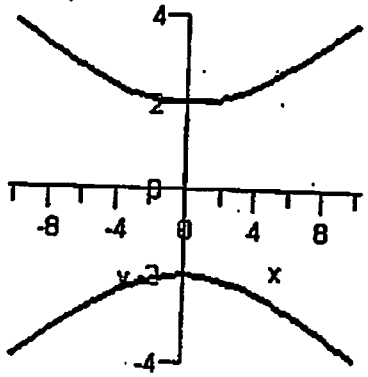
C)



D)



E)



---

31. Solve the equation of quadratic form. $(\sqrt{x})^2 - \sqrt{x} - 30 = 0$

$x - \sqrt{x} - 30 = 0$

A) $x = 5;\ x = -6$
B) $x = 36$
C) $x = \sqrt{5}i;\ x = \sqrt{6}$
D) $x = -25;\ x = 36$
E) $x = -5;\ x = 6$

let $u = \sqrt{x}$

→ $u^2 - u - 30 = 0$

$(u-6)(u+5) = 0$

$u = 6 \quad u = -5$

$\sqrt{x} = 6 \quad \sqrt{x} = -5$

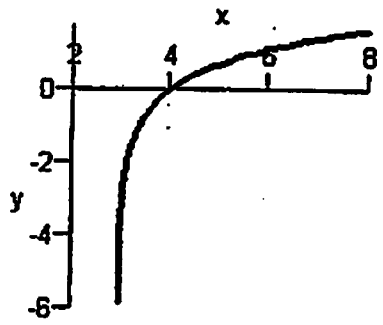
$x = 36 \quad N/A$

(B)

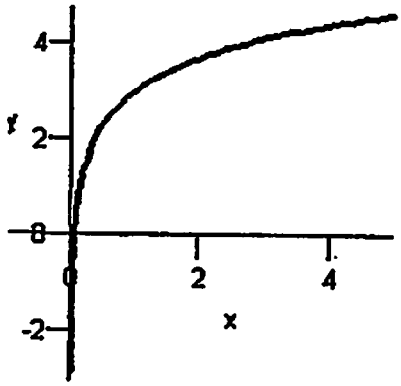
32. Graph the function $h(x)=\ln x-3$.

this is $h_0(x)=\ln x$ ⟶ x-int of $h_0(x)$ is $(1,0)$
move down 3 units ⟶ $(1,-3)$

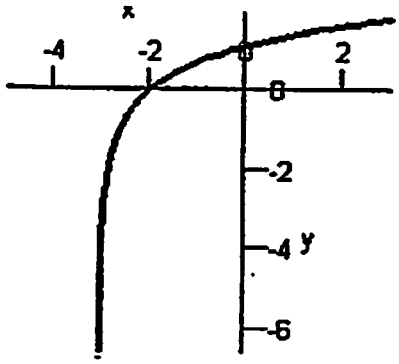
A)



B)

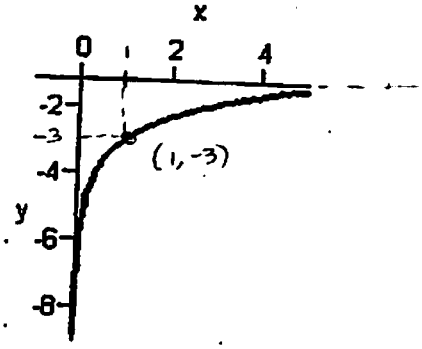


C)



D)

also $h(x)=\ln x-3$
x-int: $0=\ln x-3$
$3=\ln x$
$x=e^3$ ⟶ $(e^3,0)$



(D)

E) none of these

---

33. You and your college roommate travel to your respective hometowns in the same amount of time. You drive 200 miles and your friend drives 180 miles. Your friend's average speed is 6 miles per hour lower than your average speed. What is your average speed?

A) 58 miles per hour
B) 60 miles per hour
C) 67 miles per hour
D) 56 miles per hour
E) 64 miles per hour

Let v be your speed

$t_1 = t_2$

$\frac{200}{v} = \frac{180}{v-6}$

$200(v-6) = 180v$

$200v - 1200 = 180v$

$200v - 180v = 1200$

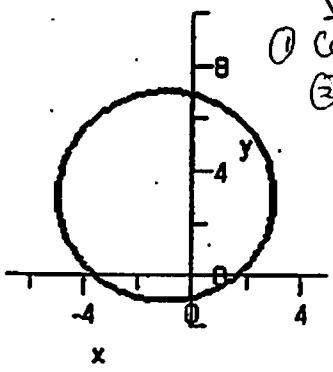
$20v = 1200$

$v = 60$

(B)

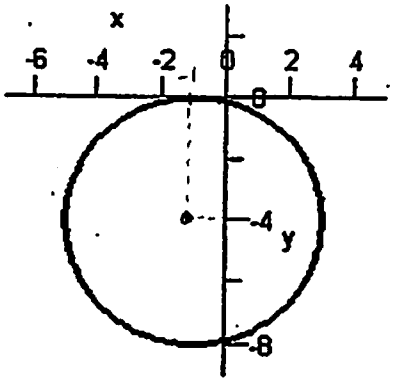
34. Graph the equation $(x+1)^2+(y+4)^2=16$.

A)

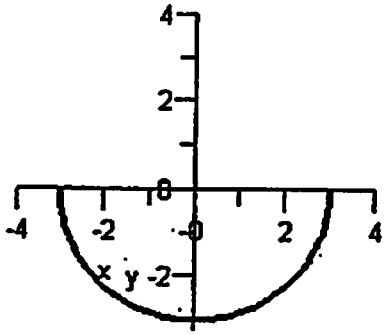


① Center $(-1,-4)$
② radius $= 4$
(B)

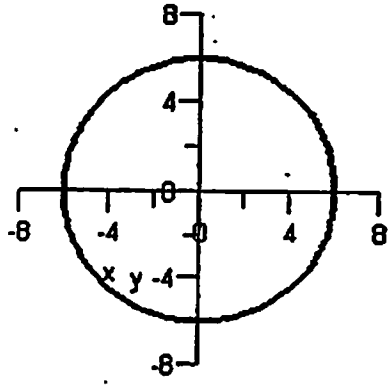
B)



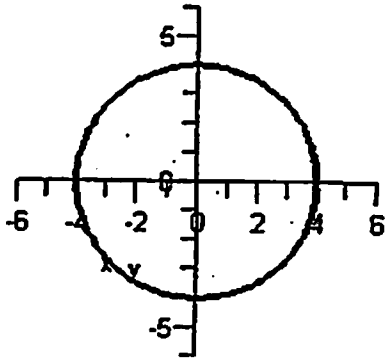
C)



D)



E)



---

35 Solve the system by the method of substitution.

$$\begin{cases} 25x^2+9y^2=225 & \cdots ① \\ 5x+3y=15 & \cdots ② \end{cases}$$

A) (3,0), (0,5)
B) (0,0)
C) no solution exists
D) (5,3)
E) (5,0), (0,3)

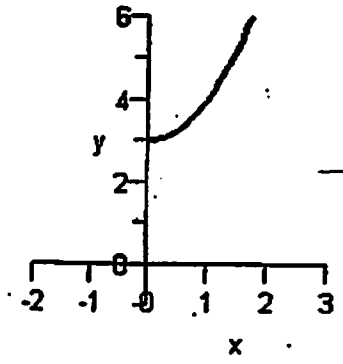
① $25x^2+9y^2=225$
$(5x)^2+(3y)^2=225$

② $5x+3y=15$
$5x=15-3y$

③ → ①: $(15-3y)^2+(3y)^2=225$
$225-90y+(3y)^2+(3y)^2=225$
$-90y+9y^2+9y^2=0$
$18y^2-90y=0$
$18y(y-5)=0$

$y_1=0,\ y_2=5$
$x_1=3\quad x_2=0$
(3,0), (0,5)
(A)

36 . Match the graph with the graph of its inverse function.



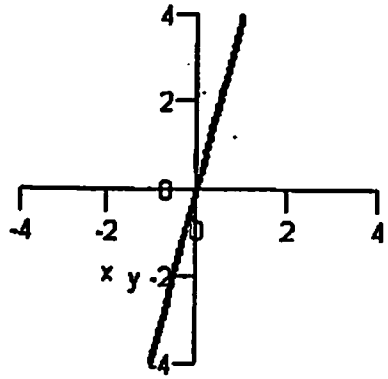
"x swaps with y" C)

→ y-int $(0,3)$

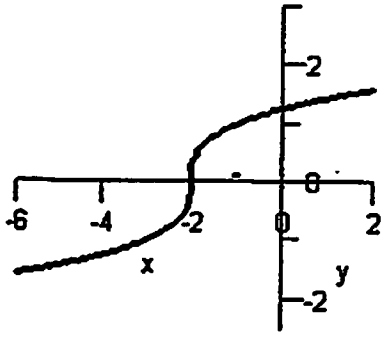
then inverse function has x-int $(3,0)$

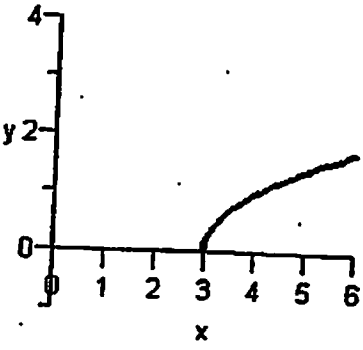
(C)

A)

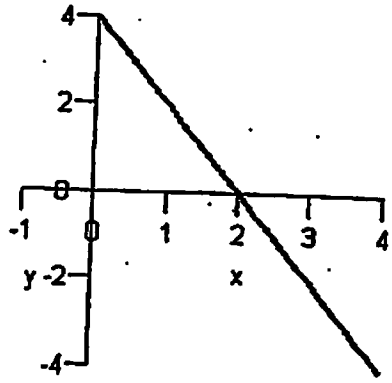


B)

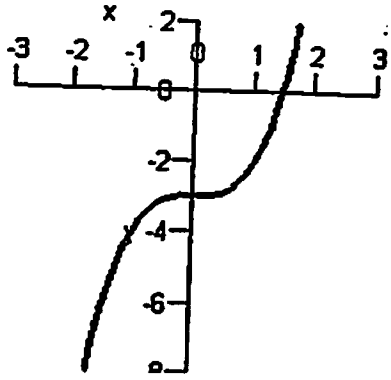




D)



E)



---

37. Solve the equation. Do not use a calculator.

$5^{x+3} = 25^5$

A) $x=1$
B) $x=4$
C) $x=7$
D) $x=2$
E) none of these

$5^{x+3} = 25^5$

$5^{x+3} = (5^2)^5$

$5^{x+3} = 5^{10}$ ← by "one-to-one" ness

→ $x+3 = 10$

$x = 7$

(C)

38. Write an equation of the line that passes through the points (5, –8) and (4, 5). Write the equation in slope-intercept form.

A) $y = 13x - 73$
B) $y = -13x - 99$
C) $y = -\frac{1}{13}x + 57$
D) $y = \frac{1}{13}x - 73$
E) $y = -13x + 57$

P_1 P_2

$m = \frac{5-(-8)}{4-5} = \frac{13}{-1} = -13$

$\longrightarrow y = -13x + b$

$P_2 \rightarrow 5 = -13(4) + b$

$5 = -52 + b$

$57 = b$

$\longrightarrow y = -13x + 57$ (E)

---

39. Evaluate the determinant of the matrix. Expand by minors along the row or column that appears to make the computation easiest.

$\begin{bmatrix} 1 & -2 & 0 \\ -2 & -2 & 2 \\ -1 & -3 & -4 \end{bmatrix}$

A) 34
B) 2
C) 22
D) 26
E) 18

$\begin{vmatrix} 1 & -2 & 0 \\ -2 & -2 & 2 \\ -1 & -3 & -4 \end{vmatrix} = 1 \begin{vmatrix} -2 & 2 \\ -3 & -4 \end{vmatrix} - (-2) \begin{vmatrix} -2 & 2 \\ -1 & -4 \end{vmatrix} = 1\left(8-(-6)\right) + 2\left(8-(-2)\right)$

↑ along First Row

$= 14 + 2 \cdot 10$

$= 14 + 20$

$= 34$

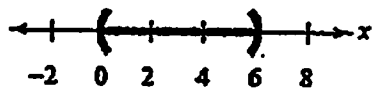
(A)

---

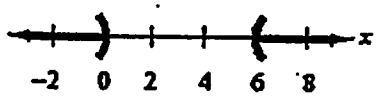
40. Graph the solution to the inequality on the real number line.

$x^2 - 6x \geq 0$

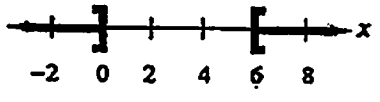
A)



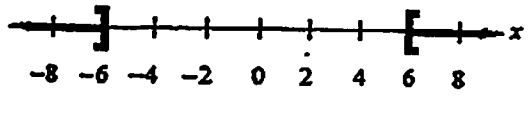
B)



C)



D)



E) none of these

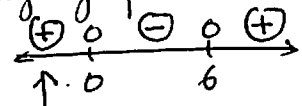
$x^2 - 6x \geq 0$

$x(x-6) \geq 0$

① Critical Numbers

$x = 0 \quad x = 6$

② Sign graph



IF $x = -1$: $(-1)[(-1)-6]$

$= (\ominus)(\ominus)$

$= \oplus \geq 0$

IF $x = 1$: $1(1-6)$

$= (\oplus)(\ominus)$

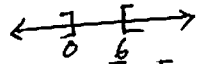
$= \ominus \leq 0$

IF $x = 7$: $7(7-6)$

$= (\oplus)(\oplus)$

$= \oplus \geq 0$

③



So, $(-\infty, 0] \cup [6, \infty)$

(C)

Fall 2011 Math 21 Test 3 Name: SOLUTION

Show all necessary work neatly, clearly, and systematically. There are 103 points available.

1. (10:2,2,1,4,1) Consider the following sample data:
4 8 5 6 4 5 7 5 → order ⇒ 4,4,5,5,5,6,7,8

Find:

a. Mean $= \frac{4+8+5+6+4+5+7+5}{8} = \frac{44}{8} = 5.5$ ($\bar{x}$)

c. Mode $= 5$

b. Median $= \frac{5+5}{2} = 5$

d. Variance $= \frac{(4-5.5)^2+(8-5.5)^2+(5-5.5)^2+(6-5.5)^2+(4-5.5)^2+(5-5.5)^2+(7-5.5)^2+(5-5.5)^2}{8-1}$

$$s^2 = \frac{(-1.5)^2+2.5^2+(-0.5)^2+0.5^2+(-1.5)^2+(-0.5)^2+1.5^2+(-0.5)^2}{7}$$

$$s^2 = \frac{2.25+6.25+0.25+0.25+2.25+0.25+2.25+0.25}{7}$$

e. Standard deviation

$s = \sqrt{2} = 1.41421$ $\quad s^2 = \frac{14}{7} = 2$

2. (16:7,2,2,1,3,1) Perform the first extension and second extension on the following frequency distribution of a sample data: (you may need only 3 columns, depending on your second extension)

x_i	freq (f_i)	$f_i x_i$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
5	2	10	-2.35	5.5225	11.045
6	3	18	-1.35	1.8225	5.4675
7	5	35	-0.35	0.1225	0.6125
8	6	48	0.65	0.4225	2.535
9	4	36	1.65	2.7225	10.89
	20	147			30.55

?? $\frac{20+1}{2} = 10.5$ → 10th, 11th

After completing the frequency distribution, answer the following:

a. mean $= \bar{x} = \frac{147}{20} = 7.35$

b. median $= \frac{7+8}{2} = 7.5$

c. mode $= 8$

d. variance $= \frac{30.55}{20-1} = \frac{30.55}{19} = 1.60789$

e. standard deviation $= \sqrt{1.60789}$

$= 1.26803$