Fall 2010 Math 125 WLAC Math Dept Final

1) $\frac{x^{\frac{1}{2}}}{x^{\frac{5}{4}} \cdot x^{-2}} = x^{\frac{1}{2}-\frac{5}{4}+2}$

$= x^{\frac{2}{4}-\frac{5}{4}+\frac{8}{4}}$

$= x^{\frac{5}{4}}$ (C)

2) $\frac{x^2}{36} + \frac{y^2}{4} = 1$

$\Rightarrow \frac{x^2}{6^2} + \frac{y^2}{2^2} = 1$

$\Rightarrow$ Ellipse, centered at (0,0)
major-vertices in x-direction
(6,0), (-6,0)
(A) and minor-vertices in
y-direction (0,2), (0,-2)

3) $x^2 + y^2 + 2x - 4y - 44 = 0$

$x^2 + 2x + y^2 - 4y = 44$

$x^2 + 2x + \underline{1} + y^2 - 4y + \underline{4} = 44 + 1 + 4$

(B) $(x+1)^2 + (y-2)^2 = 49$

circle, centered at $(-1, 2)$, $r = 7$

4) $f(x) = 3^x$

$f^{-1}(9) = x \Rightarrow f(x) = 9$

$3^x = 9$

$x = 2$ (C)

5) $2^{7+3x} = \frac{1}{4}$ $\quad \frac{1}{4} = \frac{1}{2^2} = 2^{-2}$

$\Rightarrow 2^{7+3x} = 2^{-2}$

$\Rightarrow 7 + 3x = -2$

$3x = -9$

$x = -3$ (C)

6) $f(27) = 1 + 1.5 \ln(27+1)$

$= 1 + 1.5 \ln 28$

$\cong 5.998$

(B) $\cong 6$

7) $f(x) = 4x^2 + 3x + 5$

$g(x) = 3x - 4$

$(g \circ f)(x) = g(f(x))$

$= g(4x^2 + 3x + 5)$

(C) $= 3(4x^2 + 3x + 5) - 4 = 12x^2 + 9x + 15 - 4$

8) $\log_{13} \frac{5\sqrt{r}}{s}$

$= \log_{13} 5\sqrt{r} - \log_{13} s$ (A)

$= \log_{13} 5 + \log_{13} \sqrt{r} - \log_{13} s$

$= \log_{13} 5 + \log_{13} r^{\frac{1}{2}} - \log_{13} s$

$= \log_{13} 5 + \frac{1}{2} \log_{13} r - \log_{13} s$

9) $\log_3 (15 - 3x) = 2$

exp. form $\rightarrow$ $15 - 3x = 3^2$

(B) $15 - 3x = 9$

$-3x = -6$

$x = 2$

10) $\log_{11} 11 + \log_{11} 1$

$= 1 + 0 = 1$ (C)

11) Future Value

$= \text{Principal}(1 + r)^n$

(A) $= 1000(1 + 0.10)^7$

$= 1000 \times 1.1^7 = 1,948.72$

12) $\sqrt[3]{\frac{7}{9x^2}} = \sqrt[3]{\frac{7}{3^2 x^2} \cdot \frac{3x}{3x}}$

(D) $= \sqrt[3]{\frac{21x}{3^3 x^3}}$

$= \frac{\sqrt[3]{21x}}{3x}$

13) $V \propto P$

$\rightarrow V = \frac{k}{P}$

when $V = 48$, $P = 14$

$48 = \frac{k}{14} \Rightarrow k = 48 \cdot 14$

$= 672$

when $V = 16$, $P = ?$

$16 = \frac{672}{P}$ (B)

$16P = 672$

$P = \frac{672}{16} = 42$

14) Let t be the time they work together

$\frac{t}{4} + \frac{t}{5} = 1$ $\quad \times 20$

$5t + 4t = 20$

$9t = 20$

$t = \frac{20}{9}$

(B) IF \$50/hr per employee

Cost $= \frac{20}{9} \times 50 \times 2 = 222.22$

The solution says the answer is \$111.11 because of different interpretation

15) $(3x-7)^2 + 5(3x-7) + 6 = 0$

Let $y = 3x - 7$

$\rightarrow y^2 + 5y + 6 = 0$

$(y+2)(y+3) = 0$

$(3x-7+2)(3x-7+3) = 0$ (A)

$(3x-5)(3x-4) = 0$

$3x - 5 = 0$ or $3x - 4 = 0$

$x = \frac{5}{3}$ $\quad x = \frac{4}{3}$

16) $p^2 + 11p + \underline{\quad}$ (B)

$\underline{\quad} = \left(\frac{11}{2}\right)^2 = \frac{121}{4}$

17) $p^2 + 3p - 9 = 0$

$p^2 + 3p = 9$

$\left(\frac{3}{2}\right)^2 = \frac{9}{4}$

$p^2 + 3p + \frac{9}{4} = 9 + \frac{9}{4}$

$\left(p + \frac{3}{2}\right)^2 = \frac{36}{4} + \frac{9}{4}$

$\left(p + \frac{3}{2}\right)^2 = \frac{45}{4}$

$p + \frac{3}{2} = \pm\sqrt{\frac{45}{4}} = \pm\frac{3\sqrt{5}}{2}$

(C) $p = -\frac{3}{2} \pm \frac{3\sqrt{5}}{2}$

$p = \frac{-3 \pm 3\sqrt{5}}{2}$

NOTE: Can also use quadratic formula

18) $x^2 - 4x - 4 = 0$

Can't be factorized! (A)

NOTE: Completing the Square and Quadratic Formula methods always work!

19) $(6+5i)^2$

$= 6^2 + 2 \cdot 6 \cdot 5i + (5i)^2$ (D)

$= 36 + 60i + 25i^2$

$= 36 + 60i - 25 = 11 + 60i$

20) $\frac{7-\sqrt{2}}{7+\sqrt{2}} \cdot \frac{7-\sqrt{2}}{7-\sqrt{2}} = \frac{49 - 2 \cdot 7 \cdot \sqrt{2} + 2}{49 - 2}$

(B) $= \frac{51 - 14\sqrt{2}}{47}$

21) $\frac{10}{x^2+5x} + \frac{8}{x} + \frac{2}{x+5}$

$= \frac{10}{x(x+5)} + \frac{8}{x} + \frac{2}{x+5}$ (D)

$= \frac{10 + 8(x+5) + 2x}{x(x+5)} = \frac{10 + 8x + 40 + 2x}{x(x+5)}$

$= \frac{50 + 10x}{x(x+5)} = \frac{10(x+5)}{x(x+5)} = \frac{10}{x}$

22) $\frac{4x^2+17x+23}{x^2+10x+24} - \frac{3x^2+8x+5}{x^2+10x+24}$

$= \frac{x^2+9x+18}{x^2+10x+24} = \frac{(x+3)(x+6)}{(x+4)(x+6)}$

$= \frac{x+3}{x+4}$ (B) Remember to factor!

23) $10x^3 - 360x$ (A)

$= 10x(x^2-36) = 10x(x+6)(x-6)$

24) $P = (6x+1)+(4x+20)+(x^2+6)$

$P = x^2+10x+25$

$P = (x+5)^2$ (B)

25) $y = 5x^2-35x+50$

x-int $\Rightarrow y=0 = 5x^2-35x+150$

$0 = 5(x^2-7x+10)$

$0 = 5(x-5)(x-2)$

(A) $x-5=0$ or $x-2=0$

$x=5$ $x=2$

(5,0) and (2,0)

26) $x^3 - \frac{1}{27} = x^3 - \left(\frac{1}{3}\right)^3$

(A) $= \left(x-\frac{1}{3}\right)\left(x^2 + x\left(\frac{1}{3}\right) + \left(\frac{1}{3}\right)^2\right)$

$= \left(x-\frac{1}{3}\right)\left(x^2 + \frac{1}{3}x + \frac{1}{9}\right)$

27) $\frac{x^5+x^3-3}{x-2}$

	1	0	1	0	0	-3
2		2	4	10	20	40
	1	2	5	10	20	37

(B) $\Rightarrow x^4+2x^3+5x^2+10x+20+\frac{37}{x-2}$

28) $(x-12)(x^2+5x-3)$

$= x^3+5x^2-3x$

$-12x^2-60x+36$ (B)

$= x^3-7x^2-63x+36$

29) Let p be the speed of the plane in calm air and w be the rate of the wind

$\begin{cases} 5(p+w)=1280 \\ 8(p-w)=1280 \end{cases} \Rightarrow \begin{cases} p+w=256 \\ p-w=160 \end{cases}$ (−)

$2w = 96$

$w = 48$ (C)

30) $f(x) = -(x-1)^2+4$

upside down parabola

vertex (1, 4)

axis of symmetry: $x=1$

To find x-intercepts:

$0 = -(x-1)^2+4$ (D)

$(x-1)^2 = 4$

$x-1 = \pm 2$

$x = 1 \pm 2$

$\begin{cases} x_1 = 1+2 = 3 \\ x_2 = 1-2 = -1 \end{cases} \Rightarrow \begin{matrix}(3,0)\\(-1,0)\end{matrix}$

31) $|4x-6| = 3$

$\Rightarrow 4x-6=3$ or $4x-6=-3$

$4x=9$ $4x=3$

$x=\frac{9}{4}$ (C) $x=\frac{3}{4}$

32) $\begin{vmatrix} 1 & 3 & 4 \\ -2 & 0 & 1 \\ 4 & 0 & 4 \end{vmatrix} = -3\begin{vmatrix} -2 & 1 \\ 4 & 4 \end{vmatrix}$

second column expansion

$= -3(-2\cdot4 - 4\cdot1)$

$= -3(-8-4)$

(A) $= -3(-12) = 36$

33) (−5,0), (2,6)

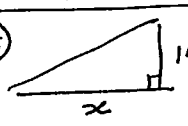
$m = \frac{6-0}{2-(-5)} = \frac{6}{7}$

$y = \frac{6}{7}x+b$, plug (−5,0)

$\Rightarrow 0 = \frac{6}{7}(-5)+b$ (A)

$0 = -\frac{30}{7}+b$

$\frac{30}{7} = b \Rightarrow y = \frac{6}{7}x+\frac{30}{7}$

34) 

$m = \frac{14}{x}$

$20\% = \frac{14}{x}$

(A) $\frac{1}{5} = \frac{14}{x}$

$x = 70$

35) ellipse $\Rightarrow$ not a function

it doesn't satisfy vertical line test

Domain $= \{x \mid -5 \le x \le 5\}$ (B)

Range $= \{y \mid -4 \le y \le 4\}$

36) $|h-9|+4 \le 9$

$|h-9| \le 5$ (D)

$-5 \le h-9 \le 5$

$4 \le h \le 14$ $[4,14]$

37) $\frac{5}{x-7} + \frac{3}{x} = -\frac{21}{x^2-7x}$

$\frac{5}{x-7} + \frac{3}{x} = -\frac{21}{x(x-7)}$ $\times x(x-7)$

$5x+3(x-7) = -21$

$5x+3x-21 = -21$

$8x-21 = -21$

$8x = 0$

$x = 0$

But, $x=0$ is not in the "domain" of the equation $\Rightarrow$ (B)

38) $f(x) = \sqrt{x-4}$

Domain $\Rightarrow x-4 \ge 0$

$x \ge 4$ (C)

Range $\Rightarrow y \ge 0$.

39) $(2x-1)^5$

$= (2x)^5 + {}_5C_1\cdot(2x)^4(-1)^1$

$+ {}_5C_2\cdot(2x)^3(-1)^2$

$+ {}_5C_3(2x)^2(-1)^3$

$+ {}_5C_4(2x)^1(-1)^4 + (-1)^5$

$= 32x^5 + 5\cdot16x^4(-1)$

$+ \cdots$

$= 32x^5 - 80x^4 \cdots$ (C)

40) $\begin{cases} x+y = -5 \\ 2x-4y-5z = -11 \\ x - z = -6 \end{cases}$

$\Rightarrow$ ① & ②, eliminate x:

$\begin{cases} x+y = -5 \\ 2x-4y-5z = -11 \end{cases} \Rightarrow \begin{cases} 2x+2y = -10 \\ 2x-4y-5z = -11 \end{cases}$ (−)

$6y+5z = 1$

$\Rightarrow$ ③ & ②

$\begin{cases} 2x-4y-5z = -11 \\ x - z = -6 \end{cases} \Rightarrow \begin{cases} 2x-4y-5z = -11 \\ 2x-2z = -12 \end{cases}$ (−)

$-4y-3z = 1$

$\Rightarrow \begin{cases} 6y+5z = 1 \\ -4y-3z = 1 \end{cases} \begin{matrix} \\ \times 2 \\ \times 3 \end{matrix} \begin{cases} 12y+10z = 2 \\ -12y-9z = 3 \end{cases}$

$z = 5$ (D)

NOTE

Questions

#14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 27, 28, 29, 33, 37 are questions doable even in Math 115!