Unknown Semester/Year
Math 125
Department Final Version 3

1) $(x+5)^{20}$

$= x^{20} + {}_{20}C_1 \cdot 5 \cdot x^{19}$
$+ {}_{20}C_2 \cdot 5^2 \cdot x^{18}$
$+ \cdots$
$= x^{20} + 20 \cdot 5 \cdot x^{19}$
$+ 190 \cdot 25 \cdot x^{18}$
$+ \cdots$
$= x^{20} + 100x^{19} + 4750x^{18}$
$+ \cdots$ (C)

2) $\begin{cases} 2x^2+y^2=17 & |\times 2| \\ 3x^2-2y^2=-6 & |\times 1| \end{cases}$

$\begin{cases} 4x^2+2y^2=34 \\ 3x^2-2y^2=-6 \end{cases}$ +

$7x^2 = 28$
$x^2 = 4$
$x = \pm 2$

→ (1): $x^2+y^2=17$
$2 \cdot 4 + y^2 = 17$
$8 + y^2 = 17$
$y^2 = 9$
$y = \pm 3$

So, $(\pm 2, \pm 3)$ D

3) $\frac{y^2}{9} - \frac{x^2}{4} = 1$

$\left(\frac{y}{3}\right)^2 - \left(\frac{x}{2}\right)^2 = 1$

→ Asymptotes: $\frac{y}{3} = \pm\frac{x}{2}$
$y = \pm\frac{3}{2}x$

Opens up-down (D)
["$y^2-x^2=1$"]

4) $\frac{x^2}{16} + \frac{y^2}{1} = 1$

$\left(\frac{x}{4}\right)^2 + \left(\frac{y}{1}\right)^2 = 1$ ellipse

· Center: (0,0)
major vertices: (4,0), (-4,0)
minor vertices: (0,1), (0,-1)

(A)

5) Circle with center (h,k) and radius r has equation
$(x-h)^2 + (y-k)^2 = r^2$
So, with center (0,2), $r=\sqrt{13}$
→ $(x-0)^2 + (y-2)^2 = \sqrt{13}^2$
(A) $x^2 + (y-2)^2 = 13$.

6) $\log(4+x) - \log(x-4) = \log 5$

$\log\left(\frac{4+x}{x-4}\right) = \log 5$

→ $\frac{4+x}{x-4} = 5$
$4+x = 5(x-4)$
$4+x = 5x-20$
$24 = 4x$
$6 = x$.

(B)

7) $2^{3x-15} = 16$

$2^{3x-15} = 2^4$

→ $3x-15 = 4$
$3x = 19$ (D)
$x = 3$

8) $V(t) = 19{,}572 \cdot 0.85^t$

$6000 = 19{,}572 \cdot 0.85^t$

$\frac{6000}{19572} = 0.85^t$ (D)

$\frac{1}{3} > 0.85^t > \frac{1}{4}$

→ $t = 7.3$ yr

8) Precise way

$V(t) = 19572 \cdot 0.85^t$

$6000 = 19572 \cdot 0.85^t$

$\frac{6000}{19572} = 0.85^t$

$\log\left(\frac{6000}{19572}\right) = \log 0.85^t$

$\log\left(\frac{6000}{19572}\right) = t \log 0.85$

$\frac{\log\left(\frac{6000}{19572}\right)}{\log 0.85} = t$ (D)

$7.275094 = t$

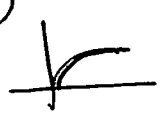
9) $f(x) = 5^x + 1$

Basic function $f_0(x) = 5^x$
· y-int: (0,1)
· horizontal asymptote y=0
· increasing

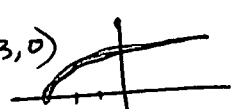
$f(x)$ is $f_0(x)$ shifted up by 1 unit. So
· y-int (0,2)
· horizontal asymptote y=1
· increasing (A)

10) $f(x) = \sqrt{x+3}$

Basic function $f_0(x) = \sqrt{x}$
x-int: (0,0)



$f(x)$ is $f_0(x)$ moved left 3 units.
So x-int (-3,0)



To find inverse:
(1) $y = \sqrt{x+3}$
(2) $x = \sqrt{y+3}$
(3) $x^2 = y+3$
(4) $x^2 - 3 = y$
$f^{-1}(x) = x^2 - 3$

(B)

Unknown semester nor year
Math 125
Department Final
Ver. 3

11) $s^z = q$

$\xrightarrow{\log} \log_s q = z$ (A)

12) $x^2 - 4x - 12 < 0$

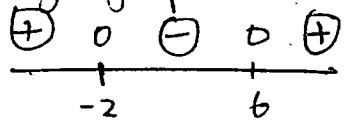
$(x-6)(x+2) < 0$

① Critical numbers.

$x-6=0 \quad x+2=0$

$x_1 = 6 \quad x_2 = -2$

② Sign graph



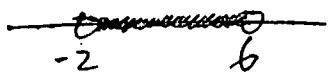
$x=0$: $(0-6)(0+2)$
$= (-)(+)$
$= (-)$

$x=10$: $(10-6)(10+2)$
$= (+)(+)$
$= (+)$

$x=-10$: $(-10-6)(-10+2)$
$= (-)(-)$
$= (+)$

③ Solution $(-2, 6)$

(C)



13) Let t be the time Tom needs, working alone, to clean the house

$$\frac{6}{8} + \frac{6}{t} = 1 \quad \times 8t$$

$6t + 48 = 8t$

$48 = 2t$

$24 = t$

Tom needs 24 hours

(A)

14) $2x - 14\sqrt{x} - 16 = 0 \quad \div 2$

$x - 7\sqrt{x} - 8 = 0$

$(\sqrt{x})^2 - 7\sqrt{x} - 8 = 0$

Let $u = \sqrt{x}$

$\rightarrow u^2 - 7u - 8 = 0$

$(u-8)(u+1) = 0$

$u-8=0$ or $u+1=0$

$u = 8 \qquad u = -1$

$\sqrt{x} = 8 \qquad \sqrt{x} = -1$

$x = 64$ — x is NOT A REAL NUMBER

(B)

15) $P = \frac{A}{1+rt}$

$P(1+rt) = A$

$P + Prt = A$

$Prt = A - P$

$r = \frac{A-P}{Pt}$ (A)

16) $2x^2 + 10x + 3 = 0$

$a=2 \quad b=10 \quad c=3$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-10 \pm \sqrt{10^2 - 4 \cdot 2 \cdot 3}}{2 \cdot 2}$$

$$= \frac{-10 \pm \sqrt{100 - 24}}{4}$$

$$= \frac{-10 \pm \sqrt{76}}{4}$$

$$= \frac{-10 \pm 2\sqrt{19}}{4}$$

$$= \frac{2(-5 \pm \sqrt{19})}{4}$$

$$= \frac{-5 \pm \sqrt{19}}{2}$$ (B)

17) $\frac{4}{7+i} \cdot \frac{7-i}{7-i}$

$= \frac{4(7-i)}{49+1}$

$= \frac{4(7-i)}{50}$ (B)

$= \frac{2(7-i)}{25} = \frac{14}{25} - \frac{2i}{25}$

18) $\sqrt{18y-27} = y+3$

$18y - 27 = (y+3)^2$

$18y - 27 = y^2 + 6y + 9$

$0 = y^2 - 12y + 36$

$0 = (y-6)^2$

(B) $0 = y - 6$

$6 = y$

NOTE: We suppose to check this solution. But, there is "no solution" option so, this answer must be the correct one

19) $\text{Time} = \frac{C}{\text{Speed}}$

For 60 sec, 5 fps

$\rightarrow C = 60 \cdot 5 = 300$

For 100 sec:

$100 = \frac{300}{\text{speed}}$ (C)

$\rightarrow$ speed = 3 fps.

20) $1 + \frac{1}{x} = \frac{12}{x^2} \quad \times x^2$

$x^2 + x = 12$

$x^2 + x - 12 = 0$

$(x+4)(x-3) = 0$ (C)

$x+4=0 \quad x-3=0$

$x_1 = -4 \quad x_2 = 3$

Unknown semester
nor year
Math 125
Department Final
Ver. 3

21) $\dfrac{1-\frac{4}{x}}{x-\frac{16}{x}} \cdot \dfrac{x}{x}$

$= \dfrac{x-4}{x^2-16}$

$= \dfrac{x-4}{(x+4)(x-4)}$

$= \dfrac{1}{x+4}$ (C)

---

22) $\dfrac{6x}{x+1} + \dfrac{7}{x-1} - \dfrac{12}{x^2-1}$

$= \dfrac{6x}{x+1} + \dfrac{7}{x-1} - \dfrac{12}{(x+1)(x-1)}$

$= \dfrac{6x(x-1)}{(x+1)(x-1)} + \dfrac{7(x+1)}{(x+1)(x-1)} - \dfrac{12}{(x+1)(x-1)}$

$= \dfrac{6x^2-6x+7x+7-12}{(x+1)(x-1)}$

$= \dfrac{6x^2+x-5}{(x+1)(x-1)}$

$= \dfrac{(6x-5)(x+1)}{(x+1)(x-1)}$

$= \dfrac{6x-5}{x-1}$ (B)

---

23) $4y^4 - 49y^2$

$= y^2(4y^2-49)$ (A)

$= y^2(2y+7)(2y-7)$

---

24) $6y^2+13y+6=0$

$(3y+2)(2y+3)=0$

$3y+2=0$ $\quad 2y+3=0$

$3y=-2$ $\quad 2y=-3$

$y=-\frac{2}{3}$ $\quad y=-\frac{3}{2}$

---

25) $y = 5x^2-35x+50$

x-int $\rightarrow y=0$

$0 = 5x^2-35x+50$ $\div 5$

$0 = x^2-7x+10$

$0=(x-5)(x-2)$

$x-5=0$ or $x-2=0$

(D) $x_1=5$ $\quad x_2=2$

x-int $(5,0)$, $(2,0)$

---

26) $\text{Area}_{\text{big}} - \text{Area}_{\text{small}}$

$=(24x^2-12x)-(16x-8)$

$=24x^2-12x-16x+8$

$=24x^2-28x+8$

$=4(6x^2-7x+2)$ (D)

$=4(3x-2)(2x-1)$

---

27) $1000c^3+27$

$=(10c)^3+3^3$

$=(10c+3)((10c)^2-(10c)3+3^2)$

(B) $=(10c+3)(100c^2-30c+9)$

---

28) $\dfrac{2x^3-5x^2-3x-34}{x-4}$

4	2	−5	−3	−34
		8	12	36
	2	3	9	2

(D)

$\rightarrow 2x^2+3x+9+\dfrac{2}{x-4}$

---

29) $(3m+9m^3)-(-9-4m-3m^3)$

$=3m+9m^3+9+4m+3m^3$

$=12m^3+7m+9$ (D)

---

30) (1) Let x be the number of adults and y be the number of senior citizens.

(2) $\begin{cases} x+y=529 \\ 23x+15y=9023 \end{cases}$

Need y, so solve for x in equation (1) before substitute

(1) $x=529-y$

(2) $23x+15y=9023$

$23(529-y)+15y=9023$

$12167-23y+15y=9023$

$12167-8y=9023$

$-8y=9023-12167$

$-8y=-3144$

$y=\dfrac{-3144}{-8}$

$y=393$ (B)

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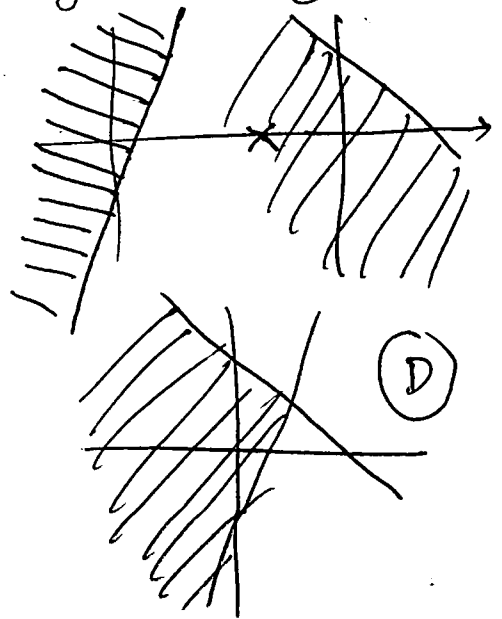
31) $\begin{cases} y \geq 2x-1 \\ x+y \leq 1 \end{cases}$

(1) $y \geq 2x-1$ (2) $x+y \leq 1$



(D)

Unknown semester / nor year
Math 125
Department Fina
Ver C

32) 3 parallel planes
→ inconsistent

33) $\begin{cases} x+3y+2z=7 \\ 3y+5z=-13 \\ z=-5 \end{cases}$

NOTE: $z=-5$

So, cross-out C, D

(3) → (2): $3y+5z=-13$

$3y+5(-5)=-13$

$3y-25=-13$

$3y=12$

$y=4$

NOTE: $z=-5$, $y=4$

→ $(\dots, 4, -5)$ (A)

34) $\begin{vmatrix} 4 & 0 & 0 \\ 8 & 5 & 7 \\ 4 & 4 & 6 \end{vmatrix}$

expand along first row

$= 4\begin{vmatrix} 5 & 7 \\ 4 & 6 \end{vmatrix} + 0\begin{vmatrix} 8 & 7 \\ 4 & 6 \end{vmatrix} + 0\begin{vmatrix} 8 & 5 \\ 4 & 4 \end{vmatrix}$

$= 4\begin{vmatrix} 5 & 7 \\ 4 & 6 \end{vmatrix} - 0 + 0$

$= 4(5\cdot 6 - 4\cdot 7)$

$= 4(30-28)$

$= 4\cdot 2$

$= 8$ (C)

35) $(-5,-8)$, $(-8,7)$

$m = \frac{7-(-8)}{-8-(-5)}$

$m = \frac{15}{-3} = -5$

→ $y = -5x + b$

$(-5,-8)$ $-8 = -5(-5) + b$

$-8 = 25 + b$

$-33 = b$

→ $y = -5x - 33$ (C)

36) $-3x-18y=18 \div 3$

$-x-6y=6$

x-int: $y=0$

→ $-x=6$

$x=-6$

$(-6,0)$

y-int: $x=0$

→ $-6y=6$

$y=-1$

$(0,-1)$ (A)

37) $f(x) = \frac{x^2-3}{x^3+7x}$

$f(2) = \frac{2^2-3}{2^3+7\cdot 2}$

$= \frac{4-3}{8+7\cdot 2}$

$= \frac{4-3}{8+14}$

$= \frac{1}{22}$ (B)

38) Not a function since it does not satisfy vertical line test.

Domain: $\{x \mid -5 \le x \le 5\}$

Range: $\{y \mid -4 \le y \le 4\}$ (D)

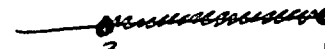
39) $|8x-1| \le 7$

→ $-7 \le 8x-1 \le 7$

$-6 \le 8x \le 8$

$\frac{-6}{8} \le x \le \frac{8}{8}$

$-\frac{3}{4} \le x \le 1$



$\left[-\frac{3}{4}, 1\right]$ (C)

40) $(y-4)-(y+3)=84$

$y-4-y-3=84$

$-7=84$

$-\frac{1}{8}=4$ (D)

NOTE

The following problems were discussed in Math 115

13, 15, 16, 18, 20, 21, 22, 23, 24, 25, 26, 29, 30, 35, 36, 37, 40

In other words, by knowing Math 115 well and no Math 125, you could answe 17 questions out of 40.