Sp'13 WLAC Math125 Dept Final

P1

1) $-2|x-2|+10=-6$

$-2|x-2|=-16$

$|x-2|=8$

$\Rightarrow x-2=8$ or $x-2=-8$

$x=10$ (B) $x=-6$

2) $|5x+1|>2$

$\Rightarrow 5x+1<-2$ or $5x+1>2$

$5x<-3$ $\quad 5x>1$

$x<-\frac{3}{5}$ (B) $\quad x>\frac{1}{5}$

3) Let x be the amount of pure water

	Amount	Concentration	Total
water	x	0	0
90% dev	9	0.9	8.1
24% dev	$x+9$	0.24	$8.1=0.24(x+9)$

$\Rightarrow 8.1=0.24(x+9)$

$8.1=0.24x+2.16$

$8.1-2.16=0.24x$

$5.94=0.24x$

$\frac{5.94}{0.24}=x$

$\frac{594}{24}=x$

$24\overline{)594}=24$, 48, 114, 96, 18

$24\frac{18}{24}=x$ (C)

$24\frac{3}{4}=x$

4) Let t be the time Chris drove

	Time	Rate	Distance
Chris	t	55	$55t$
Leslie	$12.6-t$	45	$45(12.6-t)$
	12.6		594 $=55t+45(12.6-t)$

$\Rightarrow 594=55t+45(12.6-t)$

$594=55t+567-45t$

$594-567=10t$

$27=10t$ (D)

$2.7=t$

5) $m=\frac{-1-(-6)}{9-5}=\frac{5}{4}$

$y=\frac{5}{4}x+b$

$(5,-6)\to -6=\frac{5}{4}(5)+b$

$-6=\frac{25}{4}+b$

$\to -24=25+4b$

$-49=4b$

$-\frac{49}{4}=b$ (C)

So $y=\frac{5}{4}x-\frac{49}{4}$

6) $\begin{cases} x-y+2z=-4 & (1) \\ 3x+y-4z=-6 & (2) \\ 2x+3y-4z=4 & (3) \end{cases}$

Eliminate z

(1)&(2): $\begin{cases} x-y+2z=-4 & \times 2 \\ 3x+y-4z=-6 & \times 1 \end{cases}$

$\Rightarrow \begin{cases} 2x-2y+4z=-8 \\ 3x+y-4z=-6 \end{cases}$ +

$5x-y=-14$ (a)

(2)&(3) $\begin{cases} 3x+y-4z=-6 \\ 2x+3y-4z=4 \end{cases}$ −

$x-2y=-10$ (b)

Now, (a)&(b)

$\begin{cases} 5x-y=-14 & \times(-2) \\ x-2y=-10 & \times 1 \end{cases}$ $\quad -10x+2y=28$, $x-2y=-10$ +

$-9x=18$

$x=-2$

$\Rightarrow$ (b): $x-2y=-10$

$-2-2y=-10$

$-2y=-8$

$y=4$

$\Rightarrow$ (1): $x-y+2z=-4$

$-2-4+2z=-4$

$-6+2z=-4$

$2z=2$ (A)

$z=1$

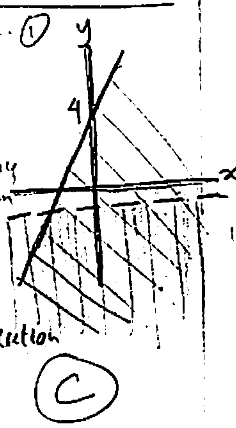
So, $(x,y,z)=(-2,4,1)$

Note

(1) I chose to eliminate z first because I think it is the easiest. However, you may choose to eliminate anyone of the variable first.

(2) Technically speaking, the solution should be (D) since A, B, and C each represents a plane while the solution is a coordinate. But, I believe the intention of the author of the test is you just need to find the true value of one variable.

7) $\begin{cases} y\le 2x+4 & \cdots (1) \\ y<-1 & \cdots (2) \end{cases}$



(1) $y\le 2x+4$

(1) $y=2x+4$ boundary

(1) $(0,0)$ is a solution

(1) $\le$ solid line

(2) $y<-1$

(2) $y=-1$ boundary

(2) $(0,0)$ is not a solution

(2) $<$ dashes

(C)

8) Let x be the unit cost of pasture fencing and y be the unit cost of picket fencing

$\begin{cases} 1800x+350y=15525 & \times 7 \\ 1400x+450y=14475 & \times 9 \end{cases}$

$\begin{cases} 12600x+2450y=108675 \\ 12600x+4050y=130275 \end{cases}$ −

$-1600y=-21600$

$y=\frac{21600}{1600}$

(A) $y=\frac{216}{16}=\frac{108}{8}=\frac{54}{4}$

$y=13.50$

9) $8x(x^2+10x+9)=0$

$8x(x+9)(x+1)=0$ (C)

$x=0$ $\quad x=-9$ $\quad x=-1$

10) $4x^2-5x-6=0$

$(4x+3)(x-2)=0$ (C)

$x=-\frac{3}{4}$ $\quad x=2$

11) $h(x)=\frac{6x}{x(x^2-25)}$

Dom $h(x)=\{x \mid x(x^2-25)\neq 0\}$

$=\{x \mid x(x+5)(x-5)\neq 0\}$

(B) $=\{x \mid x\neq 0, x\neq -5, x\neq 5\}$

12) $1-\frac{8}{x-3}=-\frac{48}{x^2-9}$

$1-\frac{8}{x-3}=-\frac{48}{(x+3)(x-3)}$ $\quad \times(x-3)(x+3)$

$(x+3)(x-3)-8(x+3)=-48$

$x^2-9-8x-24=-48$

$x^2-8x-33=-48$

$x^2-8x+15=0$ (B)

$(x-3)(x-5)=0$

$x=3$ $\quad x=5$

not in the "domain" of the equation

13) $\frac{5}{x^2-15x+54}-\frac{2}{x-6}$

$=\frac{5}{(x-6)(x-9)}-\frac{2(x-9)}{(x-6)(x-9)}$ (B)

$=\frac{5-2x+18}{(x-6)(x-9)}=\frac{-2x+23}{x^2-15x+54}$

14) $\dfrac{16 - \frac{1}{x^2}}{\frac{1}{4x^2} - 4} \cdot \dfrac{4x^2}{4x^2}$

$= \dfrac{64x^2 - 4}{1 - 16x^2} = \dfrac{4(16x^2 - 1)}{1 - 16x^2}$

$= -4$ (A)

15) $(3x^3 - x - 8) \div (x - 3)$

$$\begin{array}{r|rrrr} 3 & 3 & 0 & -1 & -8 \\ & & 9 & 27 & 78 \\ \hline & 3 & 9 & 26 & \boxed{70} \end{array}$$

$\Longrightarrow 3x^2 + 9x + 26 + \dfrac{70}{x-3}$

NOTE : This can be done also by Long-Division Method

$$\begin{array}{r} 3x^2 + 9x + 26 \\ x - 3 \overline{\smash{)}\, 3x^3 + 0x^2 - x - 8} \\ -\;\; 3x^3 - 9x^2 \\ \hline 9x^2 - x \\ -\;\; 9x^2 - 27x \\ \hline 26x - 8 \\ 26x - 78 \\ \hline 70 \end{array}$$

16) $2x^{\frac{2}{5}} + 3x^{\frac{1}{5}} + 1 = 0$

Let $y = x^{\frac{1}{5}}$

$\Longrightarrow 2y^2 + 3y + 1 = 0$

$(2y + 1)(y + 1) = 0$

$2y = -1$ $\quad$ $y = -1$

$y = -\frac{1}{2}$ $\quad$ $x^{\frac{1}{5}} = -1$

$x^{\frac{1}{5}} = -\frac{1}{2}$ $\quad$ $x = (-1)^5$

$x = \left(-\frac{1}{2}\right)^5$ $\quad$ $x = -1$

$x = -\frac{1}{32}$ (C)

17) P = price

N = number of people

$\Longrightarrow P = \dfrac{k}{N}$ or $P \cdot N = k$

\$25 , 48 people $\Rightarrow$ $25 \cdot 48 = k$

$1200 = k$

$\Rightarrow$ 71 people $\Rightarrow$ $P \cdot 71 = k$

$P \cdot 71 = 1200$

$P = \dfrac{1200}{71}$

$P = 16.90$ (D)

18) $a^{\frac{8}{3}} = (a^8)^{\frac{1}{3}} = \sqrt[3]{a^8}$ (A)

19) $f(x) = \sqrt{x+9} + 5$

Dom $f(x) = \{x \mid x + 9 \geq 0\}$

$= \{x \mid x \geq -9\}$ (A)

20) $f(x) = \sqrt{x+2}$

Dom $f(x) = \{x \mid x + 2 \geq 0\}$

$= \{x \mid x \geq -2\}$

Range $f(x) = \{y \mid y \geq 0\}$.

x-int : $0 = \sqrt{x+2}$

$0 = x + 2$

$x = -2$ $\quad$ $(-2, 0)$

y-int : $y = \sqrt{0+2}$

$y = \sqrt{2}$ $\quad$ $(0, \sqrt{2})$

(D)

21) $\sqrt{r+9} + 3 = r$

$\sqrt{r+9} = r - 3$

$r + 9 = (r-3)^2$

$r + 9 = r^2 - 6r + 9$

$0 = r^2 - 7r$

$0 = r(r - 7)$

$r = 0$, $r = 7$

Check : $r = 0 \rightarrow \sqrt{0+9} + 3 \stackrel{?}{=} 0$

$3 + 3 \stackrel{?}{=} 0$

$6 \neq 0$

$r = 7 \rightarrow \sqrt{7+9} + 3 \stackrel{?}{=} 7$

$\sqrt{16} + 3 \stackrel{?}{=} 7$

$4 + 3 \stackrel{?}{=} 7$

$7 = 7$

So, $r = 7$ (B)

22) $E = 150\sqrt{P}$

$275 = 150\sqrt{P}$

$\dfrac{275}{150} = \sqrt{P}$

$\dfrac{11}{6} = \sqrt{P}$ $\quad$ $P \approx 3.36$ (A)

$\left(\dfrac{11}{6}\right)^2 = P$

$\dfrac{121}{36} = P$

23) $\dfrac{1+4i}{10+i} \cdot \dfrac{10-i}{10-i}$

$= \dfrac{10 - i + 40i + 4}{100 + 1} = \dfrac{14 + 39i}{101}$ (B)

24) $x^2 + 2x + 17 = 0$

You don't have to use quadratic formula!

$\Longrightarrow$ They would not know anyway

$x^2 + 2x = -17$

$x^2 + 2x + 1 = -16$

$(x+1)^2 = -16$

$x + 1 = \pm\sqrt{-16} = \pm 4i$

$x = -1 \pm 4i$ (B)

If you do,

$x_{1,2} = \dfrac{-2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot 17}}{2 \cdot 1}$

$x_{1,2} = \dfrac{-2 \pm \sqrt{4 - 68}}{2}$

$x_{1,2} = \dfrac{-2 \pm \sqrt{-64}}{2}$

$x_{1,2} = \dfrac{-2 \pm 8i}{2} = \dfrac{2(-1 \pm 4i)}{2}$

$= -1 \pm 4i$

25) $x^2 - x - 56 < 0$

① Critical numbers

$x^2 - x - 56 = 0$

$(x - 8)(x + 7) = 0$

$x_1 = 8$ $\quad$ $x_2 = -7$

② Sign graph

$\oplus$ 0 $\ominus$ 0 $\oplus$

-7 $\quad$ 8

$x = 0$

(B)

③ solution $\{x \mid -7 < x < 8\}$

26) $h(t) = 128t - 16t^2$

$192 = 128t - 16t^2$

$16t^2 - 128t + 192 = 0$

$16(t^2 - 8t + 12) = 0$

$16(t - 6)(t - 2) = 0$ (C)

$t_1 = 6$ $\quad$ $t_2 = 2$

27) $(f \circ g)(x) = f(g(x))$

$= f(\sqrt{x+3})$

$= \sqrt{x+3} - 2$

(A)

28) one-to-one function
if the graph satisfies
the vertical line test
and horizontal line test

$\Longrightarrow$ i, iv (D)

29) $f(x) = 2x+4$

① $y = 2x+4$

② $x = 2y+4$

③ $x-4 = 2y$

$\frac{x-4}{2} = y$ (A)

④ $f^{-1}(x) = \frac{x-4}{2}$

30) $f(x) = 2^x - 5$

$f(0) = 2^0 - 5 = 1-5 = -4$

y-int $(0,-4)$ (D)

TO Identify further:

① base is greater than 1 with positive coefficient

$\rightarrow$ increasing

② horizontal asymptote $y=-5$

③ x-intercept

$0 = 2^x - 5$

$5 = 2^x$

$2^2 < 2^x < 2^3$

$\rightarrow$ between $x=2$ and $x=3$

④ Range : $(-5, \infty)$

31) $\log_b a = c \Leftrightarrow b^c = a$

So, $\log_6 \frac{1}{1296} = -4$ A

$\longrightarrow 6^{-4} = \frac{1}{1296}$

32) $\ln x + 7(\ln y - \ln z)$

$= \ln x + 7 \ln\left(\frac{y}{z}\right)$

$= \ln x + \ln\left(\frac{y}{z}\right)^7$

$= \ln x + \ln \frac{y^7}{z^7}$

$= \ln \frac{xy^7}{z^7}$ (A)

33) $f(x) = \log(x-6)$

Domain $= (6, \infty)$

$\longrightarrow$ no y-intercept

x-int: $0 = \log(x-6)$

$x-6 = 1$ (D)

$x = 7$

$\longrightarrow (7,0)$

34) $4^{5x-7} = 27$

$\overset{16}{4^2} < 4^{5x-7} < \overset{64}{4^3}$

$\rightarrow 2 < 5x-7 < 3$

$9 < 5x < 10$

$\frac{9}{5} < x < \frac{10}{5}$

$1.8 < x < 2$

(A)

35) $\log_9(x+3) - \log_9(x+1) = \log_9 3$

$\log_9\left(\frac{x+3}{x+1}\right) = \log_9 3$

$\rightarrow \frac{x+3}{x+1} = 3$

$x+3 = 3(x+1)$

$x+3 = 3x+3$

$x = 0$ (B)

36) $P(x) = 0.95 - 0.30 \log_2 x$

$0.35 = 0.95 - 0.30 \log_2 x$

$-0.6 = -0.30 \log_2 x$

$\frac{-0.6}{-0.3} = \log_2 x$

$2 = \log_2 x$ (B)

$x = 2^2 = 4$

37) $C(-1,-5)$

$r = 8\sqrt{2}$

$\Longrightarrow (x+1)^2 + (y+5)^2 = (8\sqrt{2})^2$

$(x+1)^2 + (y+5)^2 = 128$

(B)

38) $\frac{x^2}{49} - \frac{y^2}{25} = 1$

$\rightarrow$ ① opens along x-axis

② x-int : $(7,0), (-7,0)$

(D)

39) $\begin{cases} y = x^2 - 8x + 20 \\ x - 2y = 4 \end{cases}$

② $\Rightarrow x = 2y - 4$

② $\Rightarrow$ ① : $y = (2y-4)^2 - 8(2y-4) + 20$

$\Longrightarrow y = 4y^2 - 16y + 16 - 16y + 32 + 20$

$y = 4y^2 - 32y + 68$

$0 = 4y^2 - 33y + 68$

$0 = (4y-17)(y-4)$

$y_1 = \frac{17}{4}$ $y_2 = 4$

$x_1 = \frac{9}{2}$ $x_2 = 4$ (C)

$\rightarrow \left(\frac{9}{2}, \frac{17}{4}\right), (4,4)$

40) $(a-3b)^4$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

$\rightarrow 1a^4 - 4a^3(3b) + 6a^2(3b)^2 - 4a(3b)^3 + 1(3b)^4$

$= a^4 - 12a^3b + 6a^2 9b^2 - 4a \cdot 27b^3 + 81b^4$

$= a^4 - 12a^3b + 54a^2b^2 - 108ab^3 + 81b^4$ (D)